10/23

Monday, October 23, 2017 11:59 AM

Section 4.4

Ex: Sketch: $f(x) = x^2 - 8\ln x$

Dom: $(0, \infty)$

$$f'(x) = 2x - 8 \cdot \frac{1}{x} = 0$$

$$2x - \frac{8}{x} = 0$$

$$\frac{2x^2}{x} - \frac{8}{x} = 0$$

$$\frac{2x^2 - 8}{x} = 0$$

$$2x^2 - 8 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$x = 0$$

$$\boxed{\text{Crit pt: } x = 2}$$

$(0,2)$ $(2,\infty)$

0 2

f'	$(0,2)$	$(2,\infty)$
$2x^2-8$	$-$ 1	$+$ 3
x	$+$	$+$
f'	$-$ ↘	$+$ ↗

$\boxed{x = 2 \text{ rel. min}}$

$$f'(x) = 2x - 8x^{-1}$$

$$f''(x) = 2 + 8x^{-2}$$

$f''(x) = 2+8x^{-2}$

$\boxed{x=2 \text{ rel. min}}$

$2+\frac{8}{x^2}=0$

$\frac{2x^2}{x^2}+\frac{8}{x^2}=0$

$\frac{2x^2+8}{x^2}=0$

$2x^2+8=0$

$x^2=-4$

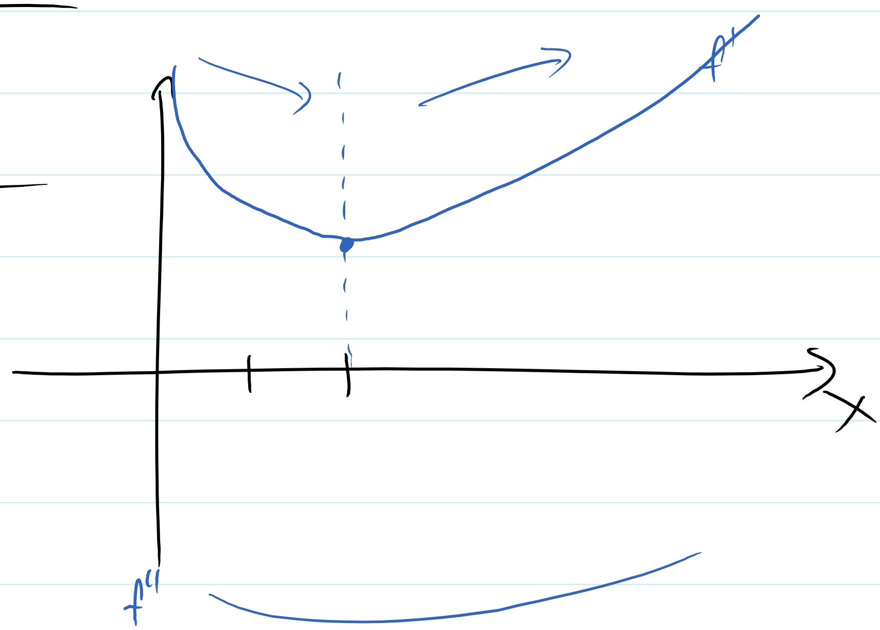
none

$x^2=0$

$x=0$

not in domain

f''	$(0,\infty)$
$2x^2+8$	$+$
x^2	$+$
	$+$



Ex:

The total number of hamburgers sold by a fast-food is growing exponentially. If 4 billion had been sold by 2005 by a fast-food is growing exponentially. If 4 billion had been sold by 2005 and 12 billion had been sold by 2010, how many will have been sold by 2015?

$$y = A \cdot B^x \qquad \left(A \cdot e^{Bx}\right)$$

Find the constants A, B

Points: $(0, 4) \longrightarrow 4 = A \cdot B^0$

$(5, 12)$ $\qquad 4 = A \cdot 1 \Rightarrow \boxed{A = 4}$

$(10, ?)$

$\boxed{y = 4 \cdot B^x}$ $\quad (5, 12) \rightarrow 12 = 4 \cdot B^5$

$$3 = B^5$$

$$\sqrt[5]{3} = B$$

$$B = 3^{1/5}$$

$$y = 4 \cdot 3^{\frac{1}{5}x} = 4 \cdot 3^{x/5}$$

$$(10, ?) \rightarrow 4 \cdot 3^{10/5} = 4 \cdot 3^2 = 4 \cdot 9 = \boxed{36}$$

Review

4.3

Use log. diff. to find y', if,

$$y = \frac{\sqrt[3]{x}\,(x^2+1)}{(x-1)^4}$$

$$\ln y = \ln\left(x^{1/3}(x^2+1)\right) - \ln(x-1)^4$$

$$\ln y = \ln x^{1/3} + \ln(x^2+1) - 4\ln(x-1)$$

$$\ln y = \frac{1}{3}\ln x + \ln(x^2+1) - 4\ln(x-1)$$

$$\frac{y'}{y} = \frac{1}{3x} + \frac{2x}{x^2+1} - \frac{4}{x-1}$$

$$y' = y\left(\frac{1}{3x} + \frac{2x}{x^2+1} - \frac{4}{x-1}\right)$$

$$y' = \boxed{\frac{\sqrt[3]{x}\,(x^2+1)}{(x-1)^4}\left(\frac{1}{3x} + \frac{2x}{x^2+1} - \frac{4}{x-1}\right)}$$

$$\boxed{\begin{aligned}(\log_b x)' &= \frac{1}{x\cdot\ln b}\\ (b^x)' &= b^x\cdot\ln b\end{aligned}}$$

- $f(x) = e^x \cdot \log_{10} x$

$$f'(x) = \log_{10} x\cdot(e^x)' + e^x(\log_{10} x)'$$

$$= \log_{10} x\cdot e^x + e^x\cdot\frac{1}{x\cdot\ln 10}$$

$$= \boxed{e^x \left(\log_{10} x + \frac{1}{x \ln 10} \right)}$$

$$(\log_b x)' = \left(\frac{\ln x}{\ln b} \right)'$$

$$= \frac{1}{\ln b} \cdot (\ln x)'$$

$$= \frac{1}{\ln b} \cdot \frac{1}{x} = \frac{1}{x \cdot \ln b}$$

- $g(x) = \sqrt[3]{\ln(x^2(x-2))}$

$$= (\ln x^2 + \ln(x-2))^{1/3}$$

$$= (2\ln x + \ln(x-2))^{1/3}$$

$$g'(x) = \boxed{\frac{1}{3} (2\ln x + \ln(x-2))^{-2/3} \cdot \underbrace{\left(\frac{2}{x} + \frac{1}{x-2} \right)}_{(2\ln x + \ln(x-2))'}}$$