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Friday, October 27, 2017 10:38 AM

Section 5.1

$f(x)$	$f'(x)$
3	0
x	1
x^3	$3x^2$
x^4	$4x^3$
$\frac{1}{3}x^3$	x^2

$f(x)$	$f'(x)$
x^{-2}	$-2x^{-3}$
x^{-1}	$-1x^{-2}$
$\frac{1}{-3}x^{-3}$	x^{-4}
$\ln x$	$x^{-1} = \frac{1}{x}$
$\frac{1}{2}x^2$	x
$x^2(x-2)^4$	$(x-2)^4 \cdot 2x + x^2 4(x-2)^3$ $= 2x(x-2)^4 + 4x^2(x-2)^3$
$\frac{-1}{11}(2-x^3)^{11}$	$3x^2(2-x^3)^{10}$

Def: A function $F(x)$ is said to be an antiderivative of $f(x)$ if

$$F'(x) = f(x)$$
$$\frac{d}{dx}F(x) = f(x),$$

for every x in the domain of $f(x)$. The process of finding antiderivatives is called process of finding antiderivatives is called antidifferentiation or indefinite integration.

Ex: Verify that $F(x)=\frac{1}{3}x^3+5x+2$ is an antiderivative of $f(x)=x^2+5$.

$$F'(x) \stackrel{?}{=} f(x)$$

$$\left(\frac{1}{3}x^3+5x+2\right)'=\frac{1}{3}\cdot 3x^2+5=x^2+5 \checkmark$$

Ex: Can you find two different antiderivatives of $f(x)=x^3+x$?

$$f(x)=x^3+x$$

$$F_1(x)=\frac{1}{4}x^4+\frac{1}{2}x^2$$

$$F_2(x)=\frac{1}{4}x^4+\frac{1}{2}x^2+21$$

Thm: (Fundamental Property of Antiderivatives)
If $F(x)$ an antiderivative of $f(x)$, then any other antiderivative of $f(x)$ has the form $G(x)=F(x)+C$ for some constant C.

note: The general antiderivative of $f(x)$ is $F(x)+C$, where

$$F'(x)=f(x)$$

notation:

$$\int f(x)\,dx = \text{"find the general antiderivative of } f(x)\text{"}$$

$$\int x^2\,dx = \frac{1}{3}x^3 + C$$

Rules for integration

- Constant rule: $\int k\,dx = k\cdot x + C$
- Power rule: $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$, $n \neq -1$.
- logarithmic rule: $\int \frac{1}{x}\,dx = \ln|x| + C$
- Exponential rule: $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

$$\boxed{\frac{d}{dx}\ln(-x) = \frac{1}{-x}\cdot(-1) = \frac{1}{x}}$$

Evaluate:

Evaluate:

- $\int 3\,dx = 3x + C$

- $\int x^{17}\,dx = \frac{1}{18}x^{18} + C$

- $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{2}{1}x^{1/2} + C$

  $= 2x^{1/2} + C$

- $\int e^{-3x}\,dx = \frac{1}{-3}e^{-3x} + C$