Section 5.2 - Integration by substitution

$$\int f'(u) \cdot u'(x)\, dx = f(u(x)) + C$$

What is the differential of $u(x)$, du?

$$du = u'(x)\, dx$$

$$\int f'(u(x)) \cdot u'(x)\, dx = \int f'(u)\, du = f(u) + C$$

Ex: $\int 18x^5 (x^6+1)^3\, dx =$ | $u = x^6+1$, $du = 6x^5\, dx$

$$= \int 3(x^6+1)^3 \cdot 6x^5\, dx = \int 3(u)^3\, du$$

$$= 3\int u^3\, du = 3 \cdot \frac{1}{4} u^4 + C$$

$$= \boxed{\frac{3}{4}(x^6+1)^4 + C}$$

Ex: $\int \sqrt{2x+7}\,dx = \int (2x+7)^{1/2}\,dx = $ $\left|\begin{array}{l} u = 2x+7 \\ \frac{du}{2} = \frac{2\,dx}{2} \\ \frac{1}{2}du = dx \end{array}\right.$

$$= \int (u)^{1/2}\,\tfrac{1}{2}\,du = \tfrac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{\cancel{2}}\,\frac{\cancel{2}}{3}u^{3/2} + C = \frac{1}{3}u^{3/2} + C = \boxed{\frac{1}{3}(2x+7)^{3/2} + C}$$

$\int 8x(4x^2-3)^5\,dx = $ $\left|\begin{array}{l} u = 4x^2-3 \\ du = 8x\,dx \\ \frac{1}{8}du = x\,dx \end{array}\right|$

$$= \frac{1}{8}\int 8(u)^5\,du = \int u^5\,du = \frac{1}{6}u^6 + C$$

$$= \boxed{\frac{1}{6}(4x^2-3)^6 + C}$$

$\int x^3 e^{x^4+2}\,dx = $ $\left|\begin{array}{l} u = x^4+2 \\ du = 4x^3\,dx \\ \frac{1}{4}du = x^3\,dx \end{array}\right.$

$$= \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C = \boxed{\frac{1}{4}e^{x^4+2} + C}$$

$$\int \frac{x}{x-1}\,dx = \left|\begin{array}{l} u = x-1 \to x = u+1 \\ du = dx \end{array}\right.$$

$$= \int \frac{u+1}{u}\,du = \int 1 + \frac{1}{u}\,du$$

$$= \int 1\,du + \int \frac{1}{u}\,du = u + \ln|u| + C$$

$$= \boxed{x - 1 + \ln|x-1| + C} = \boxed{x + \ln|x-1| + C}$$

C is <u>any</u> constant

$$\int \frac{3x+6}{\sqrt{2x^2+8x+3}}\,dx = \left|\begin{array}{l} u = 2x^2+8x+3 \\ du = (4x+8)\,dx \\ du = 4(x+2)\,dx \\ \frac{1}{4}du = (x+2)\,dx \end{array}\right|$$

$$= \int \frac{3(x+2)}{\sqrt{2x^2+8x+3}}\,dx = \frac{1}{4}\int \frac{3}{u^{1/2}}\,du$$

$$= \frac{3}{4}\int u^{-1/2}\,du = \frac{3}{4}\cdot\frac{2}{1}u^{1/2} + C$$

$$= \boxed{\frac{3}{2}\sqrt{2x^2+8x+3} + C}$$