11/03

Friday, November 3, 2017 12:05 PM

Section 5.2

Ex: $\int \frac{(\ln x)^2}{x}\,dx = \int \frac{u^2}{1}\,du$

$\left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array} \right|$

$= \int u^2\,du = \frac{1}{3}u^3 + C$

$= \boxed{\frac{1}{3}(\ln x)^3 + C}$

$\cdot \int \frac{x^2+3x+5}{x+1}\,dx$

$\left| \begin{array}{l} u = x+1 \to x = u-1 \\ du = dx \end{array} \right.$

$= \int \frac{(u-1)^2+3(u-1)+5}{u}\,du$

divide

$$\begin{array}{r} x + 2 \\ x+1 \overline{) x^2+3x+5} \\ -(x^2+x) \\ \hline 2x+5 \\ -(2x+2) \\ \hline 3 \end{array}$$

$\frac{x^2+3x+5}{x+1} = x+2+\frac{3}{x+1}$

$\int \frac{x^2+3x+5}{x+1}\,dx = \int x+2+\frac{3}{x+1}\,dx$

$= \int x+2\,dx + 3\int \frac{1}{x+1}\,dx$ $\left| \begin{matrix} u = x+1 \\ du = dx \end{matrix} \right| \int \frac{1}{u}\,du$

$= \boxed{\frac{1}{2}x^2 + 2x + 3\ln|x+1| + C}$

Ex: The price p (dollors) of each of a particular commodity is estimated to be changing at the rate

$$\frac{dp}{dx} = \frac{-135x}{\sqrt{9+x^2}},$$

where x (hundred) units is the consumer demand. Suppose 400 units are demanded when the price is \$30.

a) Find the demand function $p(x)$.

$$\begin{cases} p'(x) = \dfrac{-135x}{\sqrt{9+x^2}} \\ p(4) = 30. \end{cases}$$

$p(x) = \int \frac{-135x}{\sqrt{9+x^2}}\,dx = -135\int \frac{x}{\sqrt{9+x^2}}\,dx =$ $\left| \begin{matrix} u = 9+x^2 \\ du = 2x\,dx \\ \frac{1}{2}du = x\,dx \end{matrix} \right.$

$$= -135 \cdot \frac{1}{2} \int \frac{1}{\sqrt{u}}\, du = -\frac{135}{2} \int u^{-1/2}\, du$$

$$= -\frac{135}{\cancel{2}} \cdot \frac{\cancel{2}}{1} u^{1/2} + C = -135\sqrt{u} + C$$

$$= -135\sqrt{9+x^2} + C$$

$x^2 = 25$

$|x| = \sqrt{25}$

$|x| = 5$

$x = \pm 5$

$$p(4) = 30$$

$$-135\sqrt{9+16} + C = 30$$

$$-135 \cdot 5 + C = 30$$

$$C = 705$$

$$\boxed{p(x) = -135\sqrt{9+x^2} + 705}$$

b) what is the price when 0 units are demanded?

$$p(0) = -135\sqrt{9} + 705$$

$$= -405 + 705 = \boxed{300}$$

$$\int \frac{\overset{e^x \cdot e^x}{e^{2x}}}{1+e^x}\, dx = \quad \Bigg| \quad u = 1 + e^x \quad \Bigg| \quad e^{2x} = (e^x)^2$$

$$du = e^x\, dx \qquad = e^x \cdot e^x$$

$\hookrightarrow e^x = u - 1$

$1+e^x$

$\hookrightarrow e^x = u-1$

$$= \int \frac{u-1}{u}\,du = \int 1 - \frac{1}{u}\,du$$

$$= u - \ln|u| + C = \boxed{1+e^x - \ln|1+e^x| + C}$$

$$= 1 + e^x - \ln(1+e^x) + C$$

$\ln(A \cdot B)$
$= \ln A + \ln B$

$$= \boxed{e^x - \ln(1+e^x) + C}$$