11/08

Wednesday, November 8, 2017 12:03 PM

Section 5.3

Evaluate: $\int_0^1 e^{-x} + \sqrt{x}\, dx = \int_0^1 e^{-x} + x^{1/2}\, dx$

$= -e^{-x} + \frac{2}{3} x^{3/2} \Big|_0^1$

$= \left(-e^{-1} + \frac{2}{3} 1^{3/2}\right) - \left(-e^{-0} + \frac{2}{3} \cdot 0^{3/2}\right)$

$= -\frac{1}{e} + \frac{2}{3} + 1 - 0 = \boxed{\frac{5}{3} - \frac{1}{e}}$

Rules for def. integrals

1) $\int_a^b k f(x)\, dx = k \cdot \int_a^b f(x)\, dx$

2) Sum/diff. rule

$\int_a^b f(x) + g(x)\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$

3) $\int_a^a f(x)\, dx = 0$

4) $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

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5) Subdivision rule:

given $a<b<c$,

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

Given $\int_{-2}^{5} f(x)\,dx = 3$, $\int_{-2}^{5} g(x)\,dx = -4$, $\int_{3}^{5} f(x)\,dx = 7$

find:

a) $\int_{-2}^{5} 3f(x) - 2g(x)\,dx$

$$= 3\int_{-2}^{5} f(x)\,dx - 2\int_{-2}^{5} g(x)\,dx = 3\cdot 3 - 2\cdot(-4)$$

$$= 9 + 8 = \boxed{17}$$

$$\boxed{\int_{-\infty}^{\infty} e^{-\frac{1}{x^2}} = \frac{\pi}{2}}$$

b) $\int_{-2}^{3} f(x)\,dx$

Subdiv. rule for $-2, 3, 5$

$$\int_{-2}^{5} f(x)\,dx = \int_{-2}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$$

$$\int_{-2} f(x)\,dx = \int_{-2} f(x)\,dx + \int_{3} f(x)\,dx$$

$$3 = \int_{-2}^{3} f(x)\,dx + 7$$

$$\boxed{\int_{-2}^{3} f(x)\,dx = -4}$$

Evaluate:

$$\int_0^1 8x(x^2+1)^3\,dx$$

$u = x^2+1$ $\quad x=0 \to u = 0^2+1 = 1$

$du = 2x\,dx$ $\quad x=1 \to u = 1^2+1 = 2$

$\frac{1}{2}du = x\,dx$

$$= 8\,\frac{1}{2}\int_1^2 u^3\,du = 4\,\frac{1}{4}u^4\Big|_1^2$$

$$= u^4\Big|_1^2 = 2^4 - 1^4 = 16 - 1 = \boxed{15}$$

$\boxed{\text{OR}}$ "keep the original bounds and do back substitution"

$$= 8\cdot\frac{1}{2}\int_0^1 u^3\,du = 4\int_0^1 u^3\,du = 4\,\frac{1}{4}\,u^4\Big|_0^1$$

$$= (x^2+1)^4\Big|_0^1 = (1^2+1)^4 - (0^2+1)^4$$

$$= 2^4 - 1^4 = \boxed{15}$$

$= 2^4 - 1^4 = \boxed{15}$

$$\int_{\frac{1}{4}}^{2} \frac{\ln x}{x}\, dx = \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x}\, dx \end{array} \right.$$

$x = 2 \to u = \ln 2$

$x = \frac{1}{4} \to u = \ln \frac{1}{4}$

$$= \int_{\ln\frac{1}{4}}^{\ln 2} u\, du = \frac{1}{2} u^2 \Big|_{\ln\frac{1}{4}}^{\ln 2}$$

$$= \boxed{\frac{1}{2}(\ln 2)^2 - \frac{1}{2}\left(\ln \frac{1}{4}\right)^2}$$

$\frac{1}{4} = 2^{-2}$

$$= \frac{1}{2}(\ln 2)^2 - \frac{1}{2}(-2\ln 2)^2$$

$$= \frac{1}{2}(\ln 2)^2 - \frac{1}{2} \cdot 4\, (\ln 2)^2 = \ldots$$

Net change

If $Q'(x)$ is continuous on $[a,b]$, then the net change in $Q(x)$ as x varies between a and b is,

$$Q(b) - Q(a) = \int_a^b Q'(x)\, dx$$

Ex: ... marginal cost is $3(q-4)^2$. By how much will the total cost increase if the level of production is raised from $q=6$

much will the total cost increase if the level of production is raised from $q=6$ to $q=10$?

$$C(10)-C(6)=\int_6^{10} 3(q-4)^2\,dq$$

$$=3\int_6^{10}(q-4)^2\,dq$$

$u=q-4$ $\quad q=6 \to u=2$

$du=dq$ $\quad q=10 \to u=6$

$$=3\int_2^6 u^2\,du = 3\cdot\frac{1}{3}u^3\Big|_2^6 = u^3\Big|_2^6$$

$$=6^3-2^3=216-8=\boxed{208}$$