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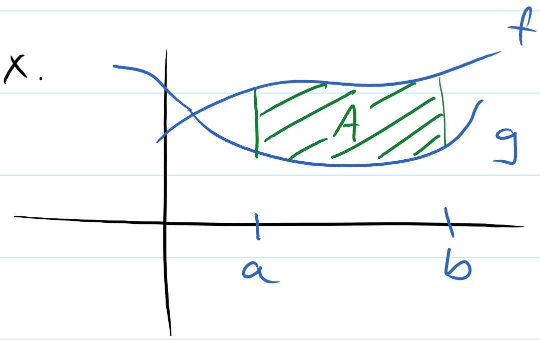
Monday, November 13, 2017 11:59 AM

Section 5.4

The area between two curves

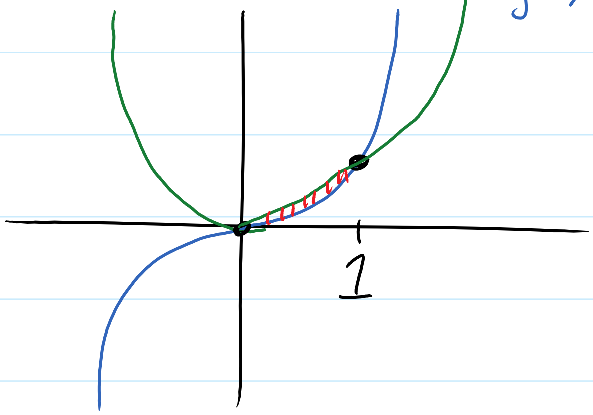
If $f(x)$ and $g(x)$ are continuous with $f(x) \geq g(x)$ on $[a,b]$, then the area A between the curves $y=f(x)$ and $y=g(x)$ over the interval is given by

$$A = \int_a^b f(x) - g(x)\, dx.$$



Ex: Find the area of the region enclosed by $y = x^3$ and $y = x^2$.

$y(\tfrac{1}{2}) = \tfrac{1}{8}$ $y(\tfrac{1}{2}) = \tfrac{1}{4}$

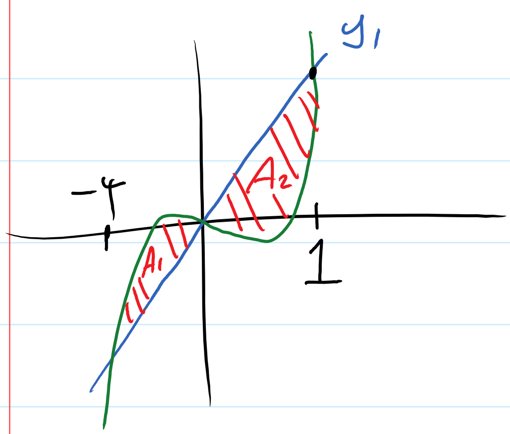


$$x^3 = x^2$$
$$x^3 - x^2 = 0$$
$$x^2(x-1) = 0$$
$$\underline{x=0}, \quad \underline{x=1}$$

$$A = \int_0^1 x^2 - x^3\, dx = \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 \Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{4} - (0 - 0) = \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

Ex: Find the area of the region enclosed by $y_1 = 4x$ and $y_2 = x^3 + 3x^2$.



$$4x = x^3 + 3x^2$$
$$0 = x^3 + 3x^2 - 4x$$
$$0 = x(x^2 + 3x - 4)$$
$$0 = x(x+4)(x-1)$$
$$x = 0, -4, 1$$

Find $A_1 + A_2$.

$$A_1 = \int_{-4}^{0} x^3 + 3x^2 - 4x \, dx = \frac{1}{4}x^4 + x^3 - 2x^2 \Big|_{-4}^{0}$$
$$= \frac{1}{4}0 + 0 - 0 - \left(\frac{1}{4}(-4)^4 + (-4)^3 - 2\cdot(-4)^2 \right)$$
$$= -(64 - 64 - 32) = \underline{32}$$

$$A_2 = \int_0^1 4x - (x^3 + 3x^2) \, dx = \int_0^1 -x^3 - 3x^2 + 4x \, dx$$
$$= -\frac{1}{4}x^4 - x^3 + 2x \Big|_0^1 = -\frac{1}{4} - 1 + 2 - (0 - 0 + 0)$$
$$= \underline{3/4}$$

Area is $\boxed{32+\frac{3}{4}} = \boxed{32.75} = \boxed{\frac{131}{4}}$

Ex: Suppose that t years from now, one investment will be generating profit at the rate of $P_1'(t) = 50+t^2$ 100s \$ per year. The second investment ... at the rate $P_2'(t) = 200+5t$.

a) For how many years does the rate of the second profit exceeds the first one?

$$50 + t^2 = 200+5t$$
$$t^2 - 5t - 150 = 0$$
$$(t-15)(t+10) = 0$$
$$t = \boxed{15}, \cancel{-10}$$

b) Complute the net excess profit for the time period from a).

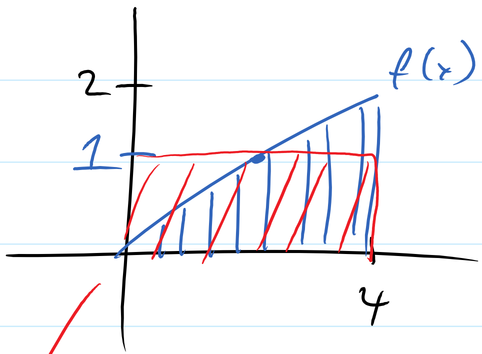
$$\int_0^{15} 200+5t-(50+t^2)\, dt$$

$$= \int_0^{15} 150+5t-t^2\, dt = 150t+\frac{5}{2}t^2-\frac{1}{3}t^3 \Big|_0^{15}$$

$$= 150 \cdot 15 + \frac{5}{2} \cdot 15^2 - \frac{1}{3} \cdot 15^3 - (0)$$

$$= \boxed{1687.5} \text{ hundred dollars}$$

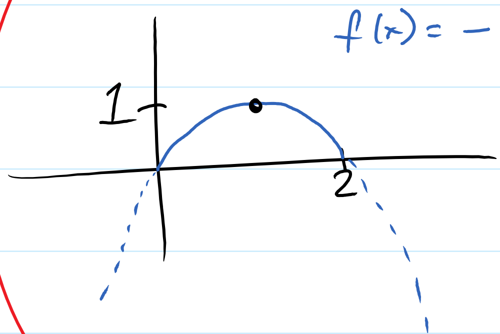
Avarage value



what is the average value of $f(x)$ on $[0, 4]$?

It is 1.

$$f(x) = -x(x-2) = -x^2 + 2x$$



what is the average on $[0, 2]$?

Area of the rect. = Area under the curve..

height × length

$f_{ave} \times (b-a)$

$$\int_a^b f(x)\,dx$$

$$f_{ave} \cdot (b-a) = \int_a^b f(x)\,dx$$

$$\boxed{f_{ave} = \frac{1}{b-a} \int_a^b f(x)\,dx}$$

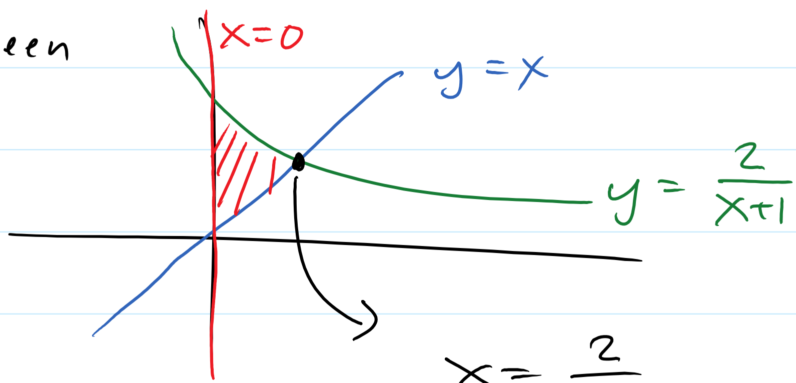
The average of $y=-x^2+2x$ on $[0,2]$:

$$\frac{1}{2-0}\cdot\int_0^2 -x^2+2x\,dx$$

$$=\frac{1}{2}\cdot\left[-\frac{1}{3}x^3+x^2\Big|_0^2=\frac{1}{2}\left[-\frac{1}{3}\cdot 2^3+2^2-0\right]\right.$$

$$=\frac{1}{2}\left[\frac{-8}{3}+4\right]=\frac{1}{2}\cdot\frac{12-8}{3}=\frac{4}{2\cdot 3}=\boxed{\frac{2}{3}}$$

Set up the integral to find the area between



$$x=\frac{2}{x+1}$$

$$x^2+x=2$$

$$x^2+x-2=0$$

$$(x-1)(x+2)=0$$

$$x=1,\cancel{-2}$$

$$\boxed{A=\int_0^1 \frac{2}{x+1}-x\,dx}$$