Section 5.4

find the average value of

$$S(t) = \frac{750t}{\sqrt{4t^2+25}} \quad \text{on } [0,6].$$

$$\frac{1}{6-0}\int_0^6 \frac{750t}{\sqrt{4t^2+25}}\,dt = \frac{750}{6}\int_0^6 \frac{t}{\sqrt{4t^2+25}}\,dt$$

$u = 4t^2+25$

$du = 8t\,dt$

$\frac{1}{8}du = t\,dt$

$$= \frac{\overset{375}{\cancel{750}}}{6\cdot\underset{4}{\cancel{8}}}\int_{\cancel{0}}^{\cancel{6}} \frac{1}{\sqrt{u}}\,du$$

$$= \frac{\overset{125}{\cancel{375}}}{\underset{2}{\cancel{6}}\cdot 4}\int_{\cancel{0}}^{\cancel{6}} u^{-1/2}\,du$$

$$= \frac{125}{\underset{4}{\cancel{8}}}\cdot\frac{\cancel{2}}{1}u^{1/2}\Big|_{\cancel{0}}^{\cancel{6}} = \frac{125}{4}\cdot\sqrt{4t^2+25}\Big|_0^6$$

$$= \frac{125}{4}\left(\sqrt{4\cdot 36+25} - \sqrt{0+25}\right)$$

$$\frac{125}{4}\left(\sqrt{\quad} - \right) = \frac{125}{4}\cdot(13-5)$$

$$= \frac{125}{4}\left(\sqrt{169} - 5\right) = \frac{125}{4}\cdot(13-5)$$

$$= \frac{125}{\cancel{4}}\cdot 8^{2} = \boxed{250}$$

Section 5.5

An annuity is a fixed sum paid someone each year.

Formula

Suppose money is being transferred continuously into an accound over a time period $0 \le t \le T$ at the rate given by a function $f(t)$ (ussualy constant) and that accound earns interest at an annual rate r compounded continuously. Then the future value (FV) of the income stream over the term T is

$$FV = \int_0^T f(t)\, e^{r(T-t)}\, dt = e^{rT}\cdot \int_0^T f(t)\, e^{-rt}\, dt$$

Ex: Saving for retirement, \$1000 per year at 5% interest for 40 years

$$FV = e^{0.05 \cdot 40} \int_0^{40} 1000\, e^{-0.05 \cdot t}\, dt$$

$$= 1000\, e^2 \int_0^{40} e^{-0.05t}\, dt$$

$$= 1000\, e^2 \cdot \frac{1}{-0.05} e^{-0.05t} \Big|_0^{40}$$

$$= \frac{1000}{-0.05} e^2 \cdot \left(e^{-0.05 \cdot 40} - e^0 \right)$$

$$= \frac{1000}{-0.05} e^2 \left(e^{-2} - 1 \right) = \frac{1000}{-0.05} \cancel{e^2 e^{-2}} + \frac{1000}{0.05} e^2$$

$$= \boxed{\frac{1000}{0.05} \left(e^2 - 1 \right)} \qquad = -1 \cdot \frac{1000}{0.05} + e^2 \cdot \frac{1000}{0.05}$$

$$= \boxed{127781.122}$$

Ex: Sue is 30 years old she starts making annual deposits of \$2000 into a fund that pays 8% annual interest compounded continuously.

interest compounded continuously.
How much money is on the account when she is 55 years old?

$T = 25$, $f(t) = 2000$

$$FV = 2000 \cdot e^{0.08 \cdot 25} \int_0^{25} e^{-0.08t}\, dt$$

$$= 2000\, e^{0.08 \cdot 25} \frac{1}{-0.08} e^{-0.08t} \Big|_0^{25}$$

$$= \frac{2000}{-0.08} e^{0.08 \cdot 25} \left(e^{-0.08 \cdot 25} - e^0 \right)$$

$e^2(e^{-2} - 1)$

$$= \frac{2000}{-0.08} \left(1 - e^2 \right)$$

$$= \boxed{\frac{2000}{0.08} (e^2 - 1)} = \boxed{159\,726.4025}$$