Section 5.1
Antiderivatives

Show that $F(x) = \frac{1}{3}x^3 - x^{-2}$ is an antiderivative of $f(x) = x^2 + \frac{2}{x^3}$

Need to differentiate $F(x)$. Show $(F(x))' = f(x)$

$$\left(\frac{1}{3}x^3 - x^{-2}\right)' = \frac{1}{3}\cdot 3x^2 - (-2)x^{-3} = \underline{x^2 + \frac{2}{x^3}}$$

Find
- $\int \frac{e^x}{2} + x\sqrt{x}\,dx = \int \frac{1}{2}e^x\,dx + \int x \cdot x^{1/2}\,dx$

$$= \frac{1}{2}\int e^x\,dx + \int x^{3/2}\,dx = \boxed{\frac{1}{2}e^x + \frac{2}{5}x^{5/2} + C}$$

- $\int \frac{1}{x}(x+1)^2\,dx$ $\quad \left|\begin{array}{l} u = x+1 \\ du = dx \end{array}\right|$ $= \int \frac{1}{u-1}u^2\,du$ ☹
$x = u-1$

$$= \int \frac{1}{x}(x^2+2x+1)\,dx = \int x + 2 + \frac{1}{x}\,dx$$
$$= \boxed{\frac{1}{2}x^2 + 2x + \ln|x| + C}$$

Section 5.3

$$\bullet \int \frac{e^{\frac{1}{x}}}{x^2}\,dx = \left| \begin{array}{l} u = \frac{1}{x} = x^{-1} \\ du = -x^{-2}\,dx \\ -du = \frac{1}{x^2}\,dx \end{array} \right| = -\int e^u\,du = -e^u + C$$

$$= \boxed{-e^{\frac{1}{x}} + C}$$

$$\bullet \int_0^1 x(x^2-3)^2\,dx = \left| \begin{array}{l} u = x^2-3 \\ du = 2x\,dx \\ \frac{1}{2}du = x\,dx \end{array} \right| = \frac{1}{2}\int_{\bullet}^{\bullet} u^2\,du$$

$$= \frac{1}{2}\cdot\frac{1}{3}\,u^3\Big|_{\bullet}^{\bullet} = \frac{1}{6}\,(x^2-3)^3\Big|_0^1 = \frac{1}{6}(1^2-3)^3 - \frac{1}{6}(0-3)^3$$

$$= \frac{1}{6}(-2)^3 - \frac{1}{6}(-3)^3 = \frac{-8}{6} + \frac{27}{6} = \boxed{\frac{19}{6}}$$

$$\bullet \int \frac{2x}{\sqrt{x^2+4}}\,dx = \left| \begin{array}{l} u = x^2+4 \\ du = 2x\,dx \end{array} \right| = \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du$$

$$= \frac{2}{1}u^{1/2} + C = \boxed{2\sqrt{x^2+4} + C}$$

HW 5.4

Q8: Find the average value $\left(\frac{1}{b-a}\int_a^b f(x)\,dx\right)$ of the function $f(x) = x^2-7x+5$ over $[-1,3]$

$$\frac{1}{3-(-1)}\int_{-1}^{3} x^2-7x+5\,dx = \frac{1}{4}\cdot\left(\frac{1}{3}x^3 - \frac{7}{2}x^2 + 5x\Big|_{-1}^{3}\right.$$

$$= \frac{1}{4}\left(\frac{1}{3}\cdot(3)^3 - \frac{7}{2}\cdot 9 + 15\right) - \frac{1}{4}\left(\frac{1}{3}(-1)^3 - \frac{7}{2}(-1)^2 + 5(-1)\right)$$

$$= \frac{1}{4}\left(9 - \frac{63}{2} + 15\right) - \frac{1}{4}\left(\frac{-1}{3} - \frac{7}{2} - 5\right)$$

$$= \frac{1}{4}\left[\underbrace{9 - \frac{63}{2} + 15}_{24} - \left(\frac{-1}{3} - \frac{7}{2} \underbrace{- 5}_{+5}\right)\right]$$

$$= \frac{1}{4}\left[29 \underbrace{- \frac{63}{2} + \frac{1}{3} + \frac{7}{2}}_{-\frac{56}{2} = -28}\right] = \frac{1}{4}\left[\underbrace{29 - 28 + \frac{1}{3}}_{\frac{4}{3}}\right] = \frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{3} = \boxed{\frac{1}{3}}$$

Q10: Two investment plans,

1) profit at the rates $P_1(t) = 60e^{0.12t}$

2) profit at the rate: $P_2(t) = 120e^{0.09t}$

a) For how many years does the second rate exeeds the first one.

$$\frac{60e^{0.12t}}{60} = \frac{120e^{0.09t}}{60}$$

$$\frac{e^{0.12t}}{e^{0.09t}} = 2\frac{e^{0.09t}}{e^{0.09t}}$$

$$e^{0.12t - 0.09t} = 2$$

$$\ln\left(e^{0.03t}\right) = \ln(2)$$

$$\frac{0.03t}{0.03} = \frac{\ln 2}{0.03} \longrightarrow t = \boxed{\frac{\ln 2}{0.03}} \approx 23.104906$$

b) Find the net excess profit

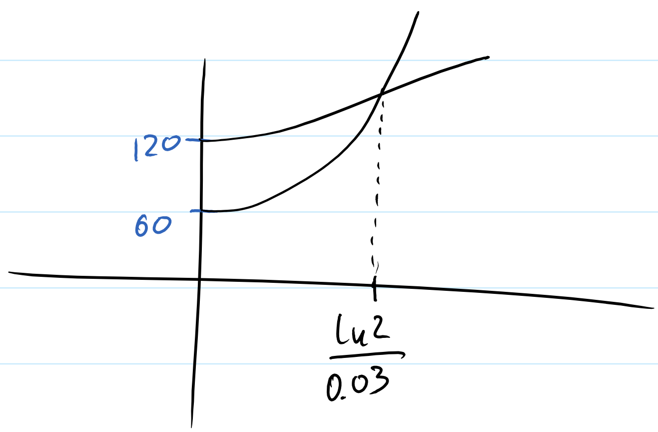
$$\int_0^{\frac{\ln 2}{0.03}} 120e^{0.09t} - 60e^{0.12t}\,dt = 120\int_0^{\frac{\ln 2}{0.03}} e^{0.09t}\,dt$$

$$\int_0^{\frac{\ln 2}{0.03}} 120e^{0.09t} - 60e^{0.12t}\,dt = 120\int_0^{\frac{\ln 2}{0.03}} e^{0.09t}\,dt - 60\int_0^{\frac{\ln 2}{0.03}} e^{0.12t}\,dt$$

$$= 120\cdot\frac{1}{0.09}e^{0.09t} - 60\frac{1}{0.12}e^{0.12t}\Big|_0^{\ln 2/0.03} \qquad \int e^{kt}\,dt = \frac{1}{k}e^{kt}$$

$$= \boxed{1833332.99}$$

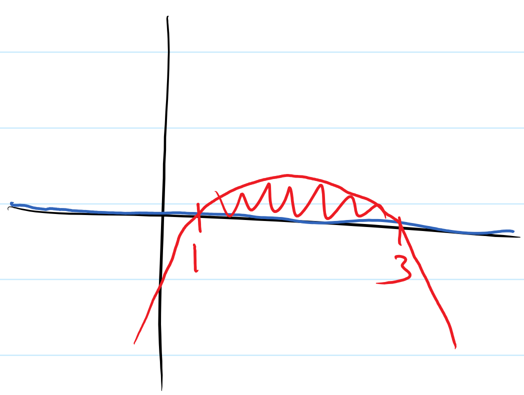
c) Plot the two functions:



Sec. 5.4

Find the area of the region bounded by the x-axis and the curve $y = -x^2 + 4x - 3$

$$-x^2+4x-3=0$$
$$-(x^2-4x+3)=0$$
$$-(x-3)(x-1)=0$$
$$x=3,1$$



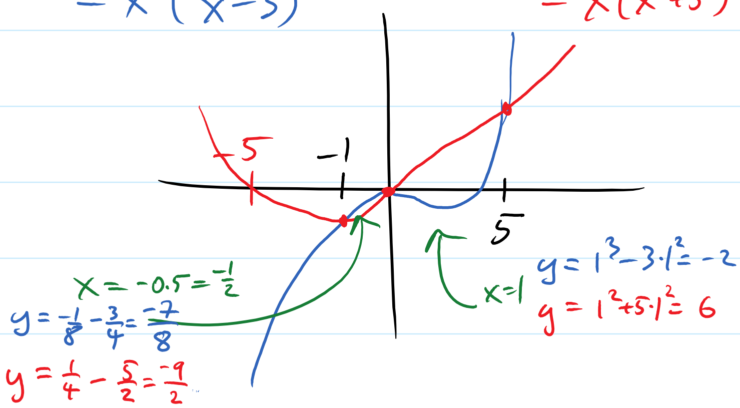
$$A = \int_1^3 -x^2+4x-3 - 0\,dx = \int_1^3 -x^2+4x-3\,dx$$

$$= -\frac{1}{3}x^3 + \frac{4}{2}x^2 - 3x \Big|_1^3 = -\frac{1}{3}\cdot 27 + 2\cdot 9 - 9 - \left(-\frac{1}{3} + 2 - 3\right)$$

$$= -\cancel{9} + \cancel{18} - \cancel{9} + \frac{1}{3} - 2 + 3 = \boxed{\frac{4}{3}}$$

- Setup the integral to find the area of the region bounded by $\underbrace{y = x^3 - 3x^2}_{= x^2(x-3)}$ and $\underbrace{y = x^2 + 5x}_{= x(x+5)}$

$$x^3 - 3x^2 = x^2 + 5x$$
$$x^3 - 4x^2 - 5x = 0$$
$$x(x^2 - 4x - 5) = 0$$
$$x(x-5)(x+1) = 0$$
$$x = 0, 5, -1$$



$$\boxed{A = \int_{-1}^{0} x^3 - 3x^2 - (x^2 + 5x)\,dx + \int_0^5 x^2 + 5x - (x^3 - 3x^2)\,dx}$$

Section 5.5

You are saving \$1200 per year for 5 years, on an account 5% interest rate comp. cont.. How much money do you have at the end?

$$FV = e^{rT}\int_0^T f(t)\, e^{-rt}\,dt$$

$r = 0.05$

$T = 5$

$f(t) = 1200$

$$FV = e^{0.05\cdot 5}\int_0^5 1200\, e^{-0.05t}\,dt$$

$$= e^{0.25} \cdot 1200 \, \frac{1}{-0.05} e^{-0.05t} \Big|_0^5$$

$$= \frac{1200}{-0.05} \, e^{0.25} \cdot \left(e^{-0.05 \cdot 5} - e^{0} \right)$$

$$= \boxed{\frac{1200}{-0.05} \left(1 - e^{0.25} \right)}$$

Given Formulas

$FV = e^{rT} \int_0^T f(t) e^{rt} \, dt$, useful life: $R'(t) = C'(t)$

You need to know:

Average value of $f(x)$ over $[a,b]$

$$\frac{1}{b-a} \int_a^b f(x) \, dx$$

Area above $f(x)$ and below $g(x)$ over $[a,b]$

$$\int g(x) - f(x) \, dx$$

Net change of $f'(x)$ between $x=a$ and $x=b$,

$$\int_a^b f'(x) \, dx$$