11/27

Monday, November 27, 2017 12:04 PM

Section 7.2

Reminder:

$$f'(x) = \frac{d}{dx} f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

But what if $f(x,y)$?

we can put the $\underline{h}$ in x or y,

$$\lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h} = f_x(x,y) = \frac{\partial}{\partial x} f$$

$$\lim_{h\to 0} \frac{f(x,y+h) - f(x,y)}{h} = f_y(x,y) = \frac{\partial}{\partial y} f$$

Def: Suppose $z = f(x,y)$. The partial derivative of f with respect to x (y) is denoted by

$$\frac{\partial z}{\partial x} \text{ or } f_x(x,y) \qquad \frac{\partial z}{\partial y} \text{ or } f_y(x,y)$$

and is the function obtained by differentiating f with respect to x (y), treating y (x) as a constant.

$\underline{\text{Ex:}}$ Find f_x, f_y of $f(x,y) = x^2 + 2xy^2 + \frac{2y}{3x}$.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^2 + 2y^2 \cdot x + \frac{2y}{3} x^{-1}\right) = 2x + 2y^2 \cdot 1 + \frac{2y}{3} \cdot (-1) x^{-2}$$

$$= \boxed{2x + 2y^2 - \frac{2y}{3x^2}}$$

$$f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^2 + 2x \cdot y^2 + \frac{2}{3x} \cdot y\right)$$

$$= 0 + 2x \cdot 2y + \frac{2}{3x} \cdot 1 = \boxed{4xy + \frac{2}{3x}}$$

$\underline{\text{Find:}}$ f_y and f_x if $f(x,y) = (x^2 + xy + y)^5$

$$f_x = 5(x^2 + xy + y)^4 \cdot \frac{\partial}{\partial x}(x^2 + yx + y)$$

$$= \boxed{5(x^2 + xy + y)^4 \cdot (2x + y)}$$

$$f_y = 5(x^2 + xy + y)^4 \cdot \frac{\partial}{\partial y}(x^2 + xy + y)$$

$$= 5(x^2 + xy + y)^4 \cdot (0 + x + 1)$$

$$= \boxed{5(x+1)(x^2 + xy + y)^4}$$

$\underline{\text{Ex:}}$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = f(x,y) = xe^{-2xy}$.

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(xe^{-2xy}\right) = x \cdot \frac{\partial}{\partial y}\left(e^{-2xy}\right)$$

chain rule

$$= x \cdot e^{-2xy} \frac{\partial}{\partial y}(\ldots) \quad \ldots e^{-2xy} (\ldots)$$

$$= x \cdot e^{-2xy} \cdot \frac{\partial}{\partial y}(-2xy) = x\, e^{-2xy} \cdot (-2x)$$

chain rule

$$= \boxed{-2x^2\, e^{-2xy}}$$

prod. rule

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(x\, e^{-2yx}\right) = \frac{\partial}{\partial x}(x) \cdot e^{-2yx} + x \cdot \frac{\partial}{\partial x}\left(e^{-2yx}\right)$$

$$= \underline{1} \cdot e^{-2yx} + x \cdot e^{-2yx} \cdot (-2yx)_x = e^{-2yx} + x\, e^{-2yx} \cdot (-2y)$$

$$= e^{-2yx} - 2xy\, e^{-2yx} = \boxed{e^{-2yx}(1 - 2xy)}$$

<u>Ex:</u> A weekly output of a factory is given by the function

$$Q(x,y) = 1200x + 500y + x^2y - x^3 - y^2,$$

where x is the # of skilled workers and y is the # of unskilled workers employed at the pant. Right now, the factory has 30 skilled workers and 60 unskilled workers. Use marginal analysis to estimate the change in the weekly output that will result from the addition of 1 more skilled worker.

Find and interpret: $Q_x(30,60)$

$$Q_x(x,y) = 1200 + 0 + y\cdot 2x - 3x^2 - 0$$
$$= 1200 + 2xy - 3x^2$$

$$Q_x(30,60) = 1200 + 2\cdot 30\cdot 60 - 3\cdot 30^2$$
$$= 1200 + 3600 - 2700$$
$$= \boxed{2100}$$

The output will increase by 2100 units.