11/29

Wednesday, November 29, 2017 10:30 AM

Section 7.2

Def: Two commodities are said to be substitute (complementary) commodities if an increase in the demand for either will result in decrease (increase) of demand for the other.

Suppose $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$ are demand functions for two commodities with prices p_1 and p_2, respectively.

Clearly,

$$\frac{\partial D_1}{\partial p_1} < 0 \quad \text{and} \quad \frac{\partial D_2}{\partial p_2} < 0.$$

Two commodities are substitute if

↳ Coke & Pepsi

$$\frac{\partial D_1}{\partial p_2} > 0 \quad \text{and} \quad \frac{\partial D_2}{\partial p_1} > 0$$

Two commodities are complementary if

↳ fries & burgers

$$\frac{\partial D_1}{\partial p_2} < 0 \quad \text{and} \quad \frac{\partial D_2}{\partial p_1} < 0.$$

Ex: Suppose $D_1(p_1, p_2) = 500 + \frac{10}{p_1+2} - 5p_2$

$$D_2(p_1, p_2) = 400 - 2p_1 + \frac{7}{p_2+3},$$

Find if the commodities are substitute, complementary or neither.

$$\frac{\partial D_1}{\partial p_2} = 0 + 0 - 5 = -5 < 0$$

$$\frac{\partial D_2}{\partial p_1} = 0 - 2 + 0 = -2 < 0$$

The commodities are complementary.

Second Partial Derivatives

If $z = f(x,y)$, the partial derivative of f_x with respect to x is

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2}.$$

Similarly:

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2},$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial x \partial y}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial y \partial x}$$

The mixed derivative of $z = f(x,y)$.

Note: $f_{yx} = f_{xy}$

Ex: Find all four second partial derivatives of

$$f(x,y) = xy^3 + 5xy^2 + 2x + 1$$

$$f_x = y^3 + 5y^2 + 2$$
$$f_y = 3xy^2 + 10xy$$

$$f_{xx} = 0$$
$$f_{yy} = 6xy + 10x$$

$$f_{yx} = \frac{\partial}{\partial x} f_y = 3y^2 + 10y$$

$$f_{xy} = \frac{\partial}{\partial y} f_x = 3y^2 + 10y$$

The same!

Ex: Find f_{xx}, f_{yy}, f_{xy} of

$$f(x,y) = \frac{x-1}{y+1}$$

$$f(x,y) = \frac{x-1}{y+1}$$

$$f_x = \frac{\partial}{\partial x}\left(\frac{1}{y+1}(x-1)\right) = \frac{1}{y+1}\frac{\partial}{\partial x}(x-1) = \frac{1}{y+1} = (y+1)^{-1}$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{x-1}{y+1}\right) = (x-1)\frac{\partial}{\partial y}\left(\frac{1}{y+1}\right) = (x-1)\frac{\partial}{\partial y}\left((y+1)^{-1}\right)$$

$$= (x-1)(-1)(y+1)^{-2}\cdot\frac{\partial}{\partial y}(y+1) = \frac{-(x-1)}{(y+1)^2}\cdot 1$$

$$= (1-x)(y+1)^{-2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left((y+1)^{-1}\right) = \boxed{0}$$

$$f_{xy} = \frac{\partial}{\partial y}\left((y+1)^{-1}\right) = -1(y+1)^{-2}\cdot\underbrace{\frac{\partial}{\partial y}(y+1)}_{=1} = \boxed{\frac{-1}{(y+1)^2}}$$

$$f_{yy} = \frac{\partial}{\partial y}\left((1-x)(y+1)^{-2}\right) = (1-x)\frac{\partial}{\partial y}\left((y+1)^{-2}\right)$$

$$= (1-x)(-2)(y+1)^{-3}\cdot 1 = \boxed{\frac{2(x-1)}{(y+1)^3}}$$

Section 7.3 – Relative max/min of a function with two variables.

Def: The point (x,y) is a critical point, if $f_x(x,y)=0$ and $f_y(x,y)=0$.