12/04

Monday, December 4, 2017 12:10 PM

Review 7.1-7.3

Ex: 7.2 119

$$f = \frac{\ln(x+2y)}{y^2}$$

$$f_x = \frac{\partial}{\partial x}\left(\frac{1}{y^2}\ln(x+2y)\right)$$

$$= \frac{1}{y^2}\cdot\left(\ln(x+2y)\right)_x = \frac{1}{y^2}\,\frac{1+0}{x+2y}$$

$$= \boxed{\frac{1}{y^2(x+2y)}}$$

$$f_y = \left(\frac{\ln(x+2y)}{y^2}\right)_y = \frac{y^2\cdot(\ln(x+2y))_y - \ln(x+2y)\cdot 2y}{y^4}$$

$$= \frac{y^2\cdot\frac{2}{x+2y} - 2y\ln(x+2y)}{y^4}$$

$$= \frac{\cancel{y}\left(\frac{2y}{x+2y} - 2\ln(x+2y)\right)}{y^{\cancel{4}3}} = \frac{\frac{2y}{x+2y} - 2\ln(x+2y)}{y^3}$$

$$= \boxed{\frac{2y - 2(x+2y)\ln(x+2y)}{(x+2y)y^3}} = \boxed{\frac{2}{(x+2y)y^2} - \frac{2\ln(x+2y)}{y^3}}$$

$\boxed{(x+2y)y^3}$ $\quad$ $\boxed{\dfrac{(x+2y)y^2}{y^3}}$

7.2 / 31

$$f = e^{x^2y}$$

$$f_x = e^{x^2y} \cdot \frac{\partial}{\partial x}(yx^2) = e^{x^2y} \cdot y \cdot 2x = \boxed{2xye^{x^2y}}$$

$$f_y = e^{x^2y} \cdot x^2 \cdot 1 = \boxed{x^2 e^{x^2y}}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(2yxe^{x^2y}\right) = 2y \frac{\partial}{\partial x}\left(xe^{x^2y}\right)$$
$$= 2y\left[1e^{x^2y} + x \cdot e^{x^2y} \cdot 2xy\right]$$
$$= \boxed{2ye^{x^2y}\left(1+2x^2y\right)}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(x^2 e^{x^2y}\right) = 2xe^{x^2y} + x^2 \cdot e^{x^2y} \cdot \frac{\partial}{\partial x}(x^2y)$$
$$= 2xe^{x^2y} + x^2e^{x^2y} \cdot 2xy = \boxed{2xe^{x^2y}\left(1+x^2y\right)}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(x^2e^{x^2y}\right) = x^2 \frac{\partial}{\partial y}\left(e^{x^2y}\right) = x^2 e^{x^2y} \cdot x^2$$
$$= \boxed{x^4 e^{x^2y}}$$

7.2 / 45

$$D_1 = \frac{7p_2}{1+p_1^2} \quad ; \quad D_2 = \frac{p_1}{1+p_2^2}$$

$$\frac{\partial D_1}{\partial p_2} = \frac{\partial}{\partial p_2}\left(\frac{7p_2}{1+p_1^2}\right) = \frac{1}{1+p_1^2} \cdot \frac{\partial}{\partial p_2}(7p_2)$$

$$= \frac{1}{1+p_1^2} \cdot 7 = \frac{7}{1+p_1^2} > 0$$

$$\frac{\partial D_2}{\partial p_1} = \frac{\partial}{\partial p_1}\left(\frac{p_1}{1+p_2^2}\right) = \frac{\partial}{\partial p_1}\left(\frac{1}{1+p_2^2} \cdot p_1\right)$$

$$= \frac{1}{1+p_2^2} \cdot \frac{\partial}{\partial p_1}(p_1) = \frac{1}{1+p_2^2} \cdot \underline{1} > 0$$

<u>Substitute</u>

$$D_1 = 2000 + \frac{100}{p_1+2} - 25p_2$$

$$D_2 = 1500 - \frac{p_2}{p_1+7}$$

$$\frac{\partial D_1}{\partial p_2} = 0 + 0 - 25 \underline{< 0}$$

$$\frac{\partial D_2}{\partial p_1} = 0 - p_2 \frac{\partial}{\partial p_1}\left(\frac{1}{p_1+7}\right) = -p_2 \frac{\partial}{\partial p}\left((p_1+7)^{-1}\right)$$

$$= -p_2 \cdot (-1)(p_1+7)^{-2} \cdot (1+0) = \frac{p_2}{(p_1+7)^2} \underline{> 0}$$

Neither

7.3/6

Find min/max/saddle pt

$$f = xy + \frac{8}{x} + \frac{8}{y} = xy + 8x^{-1} + 8y^{-1}$$

$$f_x = y + 8\cdot(-1)\,x^{-2} = 0 \rightarrow \left\{\begin{array}{l} y - \frac{8}{x^2} = 0 \\ x - \frac{8}{y^2} = 0 \end{array}\right.$$

$$f_y = x + 8(-1)\,y^{-2} = 0 \rightarrow$$

$$y = \frac{8}{x^2}$$

$$x - \frac{8}{\frac{8^2}{x^4}} = 0$$

$$y = \frac{8}{2^2} = 2$$

$$x - \frac{\cancel{8}x^4}{8^{\cancel{2}}} = 0$$

$$x\left(1 - \frac{x^3}{8}\right) = 0$$

$$\cancel{x = 0} \qquad \frac{x^3}{8} = 1$$

not in D

$\cancel{x=0}$ not in D $\quad \frac{x}{8} = 1$

$x = 2$

crit: (2, 2)

$f_{xx} = 0 + 8(-1)(-2)x^{-3} = \frac{16}{x^3}$

$f_{yy} = 0 + 8(-1)(-2)y^{-3} = \frac{16}{y^3}$

$f_{xy} = 1$

$$D = f_{xx} f_{yy} - (f_{xy})^2$$
$$= \frac{16^2}{x^3 y^3} - 1 \quad = \frac{256}{x^3 y^3} - 1$$

$D(2,2) = \frac{256}{8 \cdot 8} - 1 > 0$

$f_{xx}(2,2) = \frac{16}{8} > 0$

$\boxed{(2,2) \text{ is a rel. min.}}$