8/23

Wednesday, August 23, 2017 12:02 PM

Review:

2 $\left(\dfrac{2x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

$$= \left(\frac{2\cdot y^{7/2}}{x^{1/2}}\right)^{-2} = \left(\frac{x^{1/2}}{2y^{7/2}}\right)^{2}$$

$$= \frac{x^{\frac{1}{2}\cdot 2}}{2^2\cdot y^{7/2\cdot 2}} = \boxed{\frac{x}{4y^7}}$$

$$\frac{x^3}{x^2} = \frac{x^{3-2}}{1} = \frac{x}{1}$$

$$\frac{x^3}{x^2} = \frac{1}{x^{2-3}} = \frac{1}{x^{-1}}$$

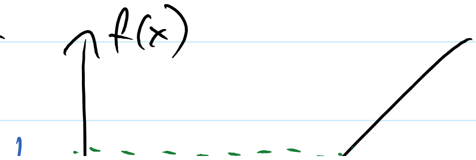
$$\frac{x^{3/2}}{x^2} = \frac{1}{x^{2-3/2}} = \frac{1}{x^{1/2}}$$

$$\frac{y^3}{y^{-1/2}} = \frac{y^{3-(-1/2)}}{1} = \frac{y^{3.5}}{1}$$

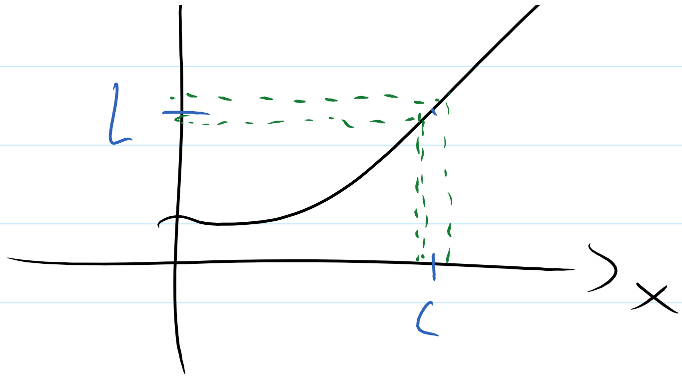
Limits, 1.5

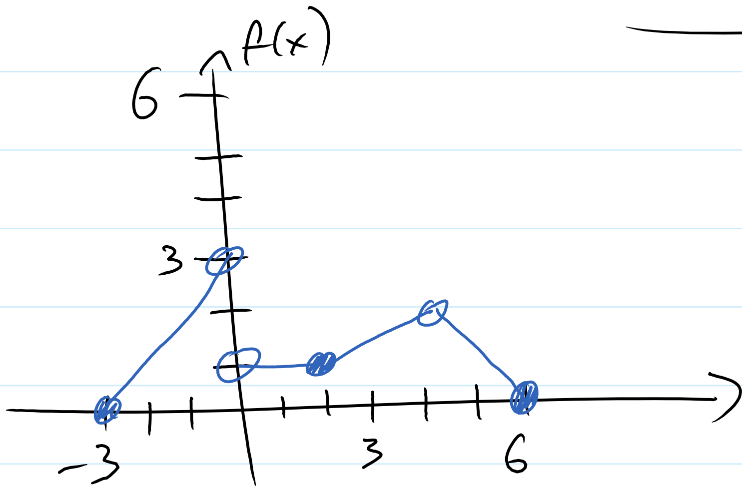
Definition: If $f(x)$ gets closer and closer to a number L as x get closer and closer to c from both sides, then L is the limit of $f(x)$ as x approaches c. Written as

$$\lim_{x\to c} f(x) = L$$



$x \to c$





$$\lim_{x \to 2} f(x) = 1$$

$$\lim_{x \to 4} f(x) = 2$$

$f(4)$ DNE

$f(0)$ DNE

$\lim_{x \to 0} f(x)$ DNE b/c we approach diff values

Ex: $\displaystyle\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$ $\qquad \dfrac{\sqrt{1}-1}{1-1} = \dfrac{0}{0}$ DNE

x	0.9	0.95	0.99	1	1.01	1.05	1.1
$\dfrac{\sqrt{x}-1}{x-1}$	0.513	0.506	0.501	X	0.498	0.493	0.488

the function values approach 0.5

Limit properties

If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist, then

- $\lim_{x\to c} [f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$

k, p constants

- $\lim_{x\to c} [k \cdot f(x)] = k \cdot \lim_{x\to c} f(x)$
- $\lim_{x\to c} [f(x) \cdot g(x)] = \left(\lim_{x\to c} f(x)\right) \cdot \left(\lim_{x\to c} g(x)\right)$
- $\lim_{x\to c} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, if $\lim_{x\to c} g(x) \neq 0$.
- $\lim_{x\to c} [f(x)]^p = [\lim_{x\to c} f(x)]^p$, if $[\lim_{x\to c} f(x)]^p$ exists

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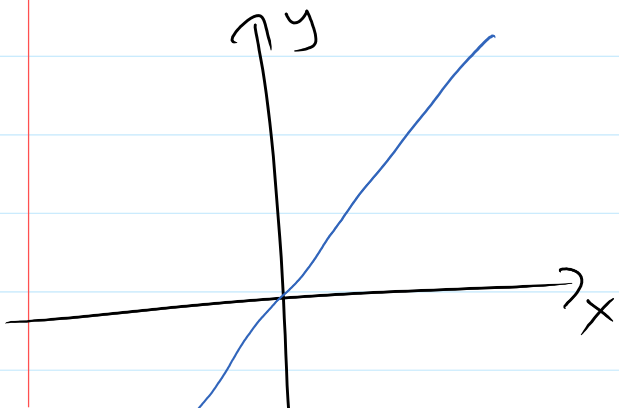
Ex:

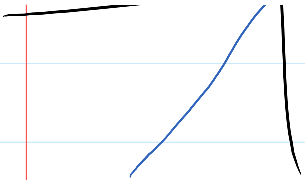
$f(x) = x$



$\lim_{x\to 8} x = 8$

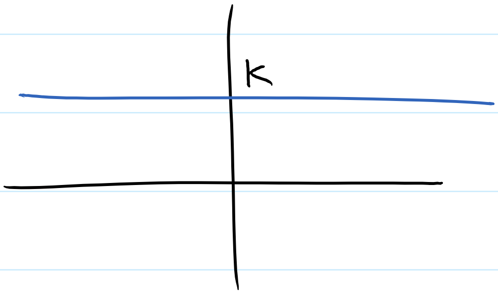
$\lim_{x\to -3} x = -3$

$\boxed{\lim_{x\to c} x = c}$



$$\boxed{\lim_{x\to c} x = c}$$

Ex: $f(x) = k$, k a constant



$$\lim_{x\to 7} k = k$$

$$\boxed{\lim_{x\to c} k = k}$$

Ex:

$$\lim_{x\to -1}(3x^3 - 4x + 8) = \lim_{x\to -1}(3x^3) - \lim_{x\to -1}(4x) + \lim_{x\to -1} 8$$

$$= 3\cdot\lim_{x\to -1}(x^3) - 4\lim_{x\to -1} x + 8$$

$$= 3\cdot\left[\lim_{x\to -1} x\right]^3 - 4\cdot(-1) + 8$$

$$= 3\cdot(-1)^3 + 4 + 8 = -3 + 12 = \boxed{9}$$

$x = -1$: $3(-1)^3 - 4(-1) + 8 = -3 + 12 = \boxed{9}$

① To find a limit of a polynomial, evaluate the polynomial.

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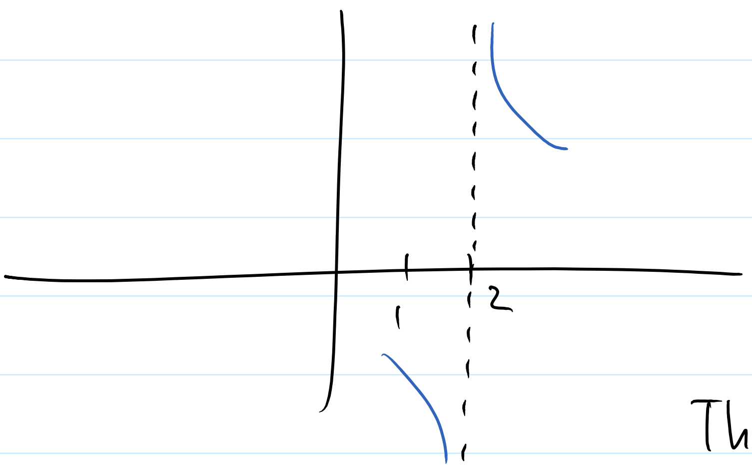
Ex:
$$\lim_{x\to 1} \frac{3x^3-8}{x-2} = \boxed{5} \qquad \frac{3(1)^3-8}{1-2} = \frac{-5}{-1} = 5$$

$$\lim_{x\to 2} \frac{3x^3-8}{x-2} \qquad \frac{3\cdot 2^3-8}{2-2} = \frac{3\cdot 8-8}{0} = \frac{16}{0} \ \underline{\text{DNE}}$$

$$f(x) = \frac{3x^3-8}{x-2}$$



$f(2.1) = 197.83$
$f(1.9) = -125.77$

The limit DNE since the function doesn't approach the same value.

Ex: $$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)} = \lim_{x\to 1}\frac{1}{\sqrt{x}+1}$$
$$= \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

Ex: $$\lim_{x\to 1} \frac{x^2-1}{x^2-3x+2} \qquad \frac{1-1}{1-3+2} = \frac{0}{0}$$ Do algebra!

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x-2)} = \lim_{x \to 1} \frac{x+1}{x-2}$$

$$= \frac{1+1}{1-2} = \frac{2}{-1} = \boxed{-2}$$