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Friday, August 25, 2017 11:59 AM

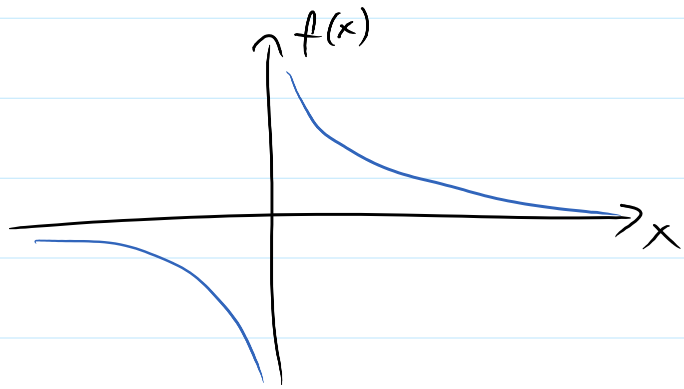
Definition

If the values of the function $f(x)$ approach the number L as x increases / decreases without bound, we write

$$\lim_{x \to \infty} f(x) = L$$

$$\lim_{x \to -\infty} f(x) = L$$

$$f(x) = \frac{1}{x}$$



$$\boxed{\lim_{x \to \pm\infty} \frac{1}{x} = 0}$$

Ex:

$$\lim_{x \to \infty} \frac{2x^2-3}{x^2+4x-1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \to \infty} \frac{\frac{2x^2}{x^2} - \frac{3}{x^2}}{\frac{x^2}{x^2} + \frac{4x}{x^2} - \frac{1}{x^2}}$$

$$= \frac{\lim_{x \to \infty} 2 - \frac{3}{x^2}}{\lim_{x \to \infty} 1 + \frac{4}{x} - \frac{1}{x^2}} = \frac{\lim_{x \to \infty} 2 - 3\left(\lim_{x \to \infty} \frac{1}{x}\right)^2}{\lim_{x \to \infty} 1 + 4 \lim_{x \to \infty} \frac{1}{x} - \left(\lim_{x \to \infty} \frac{1}{x}\right)^2}$$

$$= \frac{2 - 3 \cdot 0^2}{1 + 4 \cdot 0 - 0^2} = \frac{2}{1} = \boxed{2}$$

$$= \frac{2}{1+4\cdot 0-0^2} = \frac{2}{1} = \boxed{2}$$

Ex:

$$\lim_{x\to-\infty} \frac{x^2}{1+x+x^2-x^3}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to-\infty}\frac{\underbrace{\frac{1}{x}}_{\to 0}}{\underbrace{\frac{1}{x^3}}_{0}+\underbrace{\frac{1}{x^2}}_{0}+\underbrace{\frac{1}{x}}_{0}-1}$$

$$= \frac{0}{0+0+0-1} = \frac{0}{-1} = \boxed{0}$$

$$\lim_{x\to 0}\frac{1}{x^2} = \infty$$

Def: We say that $\lim_{x\to c} f(x)$ is an infinite limit if $f(x)$ increases / decreases without bound as $x\to c$. We write

$$\lim_{x\to c} f(x) = \infty \qquad \lim_{x\to c} f(x) = -\infty$$

A company estimates the profit function,

$$P(x) = 4x - \sqrt{x}.$$

Find out what the average profit approaches

$P(x) = 4x - \sqrt{x}$.

Find out what the average profit approaches as $x \to 0$.

average profit

$$AP(x) = \frac{P(x)}{x} = \frac{4x - \sqrt{x}}{x}$$

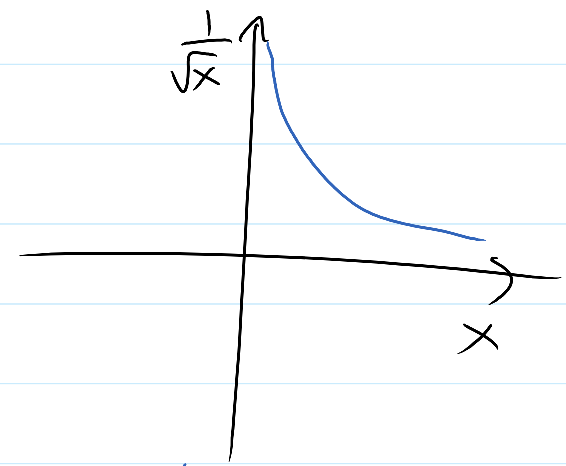
$$\lim_{x \to 0} \frac{4x - \sqrt{x}}{x} \qquad \boxed{\frac{0}{0}} \text{ do algebra}$$

$$\frac{\sqrt{x}}{x} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\cancel{x}}{\cancel{x}\sqrt{x}} = \frac{1}{\sqrt{x}}$$

$$= \lim_{x \to 0} \frac{4\cancel{x}}{\cancel{x}} - \frac{\sqrt{x}}{x} = \lim_{x \to 0} 4 - \frac{1}{\sqrt{x}}$$

$$= 4 - \lim_{x \to 0} \frac{1}{\sqrt{x}}$$

$$= 4 - \infty = \boxed{-\infty}$$



$\underbrace{\infty \text{ algebra}}$

$\infty + \# = \infty$

$\infty - \# = \infty$

$\infty + \infty = \infty$

$-\infty \pm \# = -\infty$

$-\infty - \infty = -\infty$

$\underbrace{\infty \cdot \infty = \infty}$

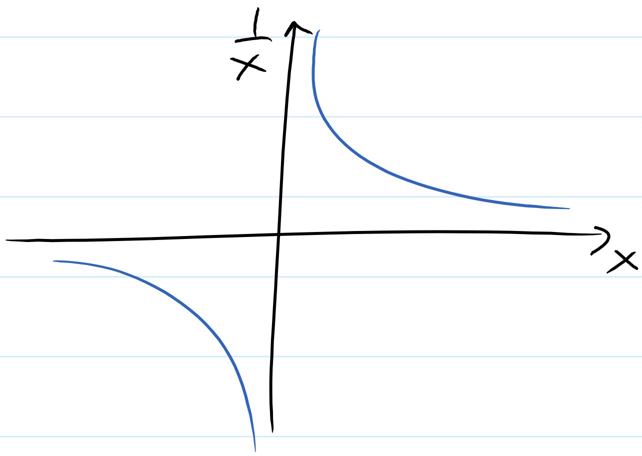
Not defined:

$\infty - \infty$

$\frac{\infty}{\infty}$

Section 1.6

One-sided limits.

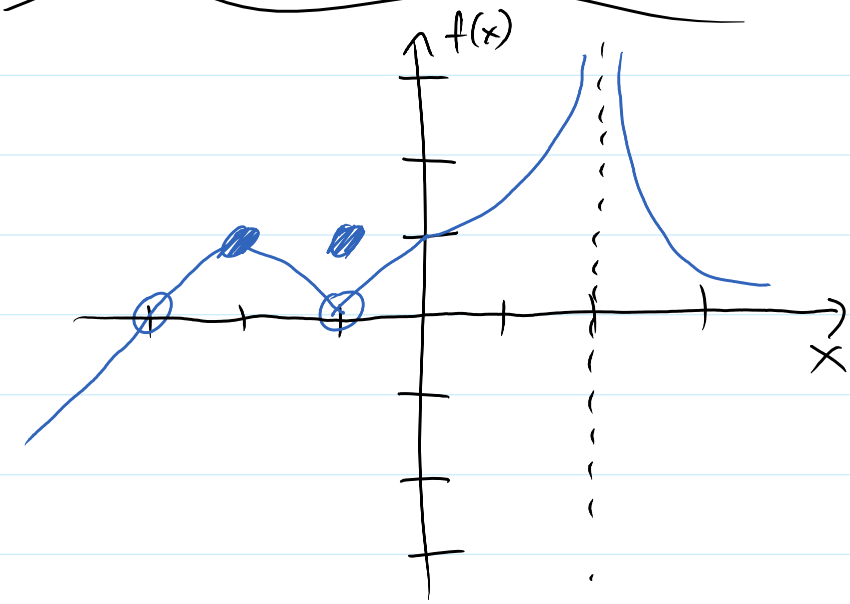


right hand side

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

left hand side limit



$$\lim_{x \to -3^-} f(x) = 0$$

$$\lim_{x \to -3^+} f(x) = 0$$

$$\lim_{x \to -3} f(x) = 0$$

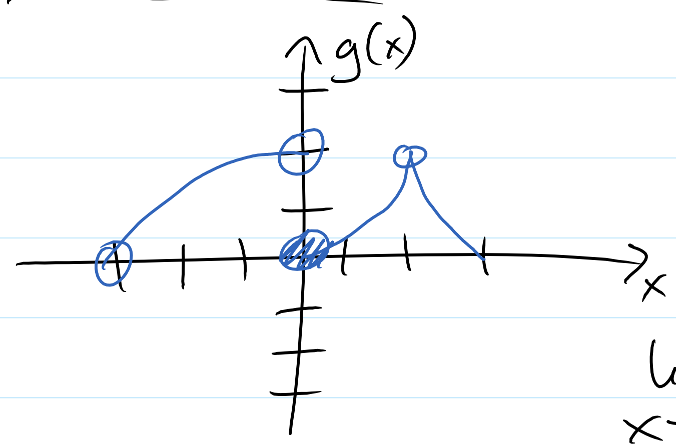
$$\lim_{x \to 2^-} f(x) = \infty$$

$$\lim_{x \to 0^+} f(x) = 1 = \lim_{x \to 0^-} f(x)$$

Theorem:

$$\lim_{x \to c} f(x) = L \text{ if and only if } \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

$\lim_{x\to c} f(x) = L$ if and only if $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$



$\lim_{x\to 0^-} g(x) = 2$

$\lim_{x\to 0^+} g(x) = 0$

$\lim_{x\to 0} g(x)$ DNE b/c left hand side and right hand side limits are different.

Ex:

$$f(x) = \begin{cases} x+1, & x<1 \\ -x^2+4x-1, & x \geq 1 \end{cases}$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} -x^2+4x-1 = -(1)^2+4\cdot 1-1 = -1+4-1 = \boxed{2}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x+1 = 1+1 = \boxed{2}$$

$$\lim_{x\to 1} f(x) = \boxed{2}$$

Ex: $g(x) = \dfrac{x-2}{x-4}$

$$\frac{4^+-2}{4^+-4} = \frac{2}{0^+} \to +\infty$$

$4.1-4$

$$\lim_{x\to 4^+} \frac{x-2}{x-4} = \boxed{+\infty}$$

$4.1-4$
0.1

$$\lim_{x\to 4^-} \frac{x-2}{x-4} \;``=\; \frac{4-2}{4^- - 4} = \frac{2}{0^-}" = \boxed{-\infty}$$

$3.9-4$
-0.1