8/28

Monday, August 28, 2017 11:58 AM

Section 1.6

$$\lim_{x\to -1} \frac{1-x^2}{x+1} = \lim_{x\to -1} \frac{(1-x)\cancel{(1+x)}}{\cancel{x+1}} \qquad \frac{1-1}{-1+1} = \frac{0}{0}$$

$$= \lim_{x\to -1} \frac{1-x}{1} = \frac{1-(-1)}{1} = \frac{2}{1} = \boxed{2}$$

$$\lim_{x\to 1} \frac{1-x^2}{x-1} = \lim_{x\to 1} \frac{(1-x)(1+x)}{-(-x+1)} = \lim_{x\to 1} \frac{\cancel{(1-x)}(1+x)}{-\cancel{(1-x)}}$$

$$= \lim_{x\to 1} \frac{1+x}{-1} = \frac{1+1}{-1} = \boxed{-2}$$

$$\lim_{x\to\infty} \frac{2x^3-x+5}{x^2-4x} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{2x - \cancel{\frac{1}{x}}^{\to 0} + \cancel{\frac{5}{x^2}}^{\to 0}}{1 - \cancel{\frac{4}{x}}_{\to 0}}$$

$$= \frac{2\cdot\infty}{1} = \boxed{\infty}$$

Online quiz 1

- due next week
- 10 question, each can be answered 3 times
- 60 minutes.
- on HW1, HW2.

Ex: $f(x) = \begin{cases} x+1, & x<1 \\ \ldots \end{cases}$

Ex: $f(x) = \begin{cases} x+1, & x<1 \\ -x^2+4x-1, & x \geq 1 \end{cases}$

$$\lim_{x \to 1} f(x) = \boxed{2}$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x+1 = 1+1 = \boxed{2}$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} -x^2+4x-1 = -1^2+4 \cdot 1 - 1 = \boxed{2}$$

Def: A function f is continuous at $x=c$ if all three of these conditions are satisfied:

a) $f(c)$ is defined

b) $\lim_{x \to c} f(x)$ exists

c) $f(c) = \lim_{x \to c} f(x)$.

If $f(x)$ is not continuous at $x=c$, we say "$f(x)$ is discontinuous at $x=c$".

Ex: Show that $p(x) = 3x^3 - x + 5$ is continuous at $x=1$

Ex: Show that $p(x) = 3x^3 - x + 5$ is continuous at $x=1$.

$$p(1) = 3\cdot 1 - 1 + 5 = 7$$

$$\lim_{x\to 1} p(x) = \lim_{x\to 1} 3x^3 - x + 5 = 3\cdot 1 - 1 + 5 = 7$$

$p(x)$ is continuous at $x=1$.

Note: All polynomials are continuous for all x.

Ex: is $f(x) = \dfrac{x+1}{x-2}$ continuous at $x=3$?

$$\left.\begin{aligned} &f(3) = 4 \\ &\lim_{x\to 3} \frac{x+1}{x-2} = \frac{3+1}{3-2} = 4 \end{aligned}\right\} f(x) \text{ is cts at } x=3.$$

Is it cts at $x=2$? No $f(2)$ DNE.

Is $g(x) = \dfrac{x^2-1}{x+1}$ continuous for all x?

$$\lim_{x\to -1} \frac{x^2-1}{x+1} = \lim_{x\to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = -1-1 = -2$$

$g(-1) = \frac{0}{0}$ not defined!

g is discontinuous at $x=-1$.

Ex: For what value of A is the function continuous for all x?

$$f(x)=\begin{cases} Ax+5, & x<1 \\ x^2-3x+4, & x\geq 1 \end{cases}$$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)$$

$$A\cdot 1+5 = 1^2-3\cdot 1+4$$

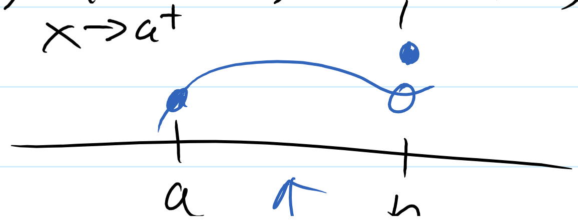
$$A+5 = 2$$

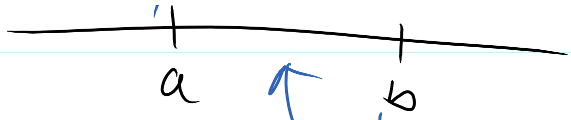
$$\boxed{A=-3}$$

Def: A function $f(x)$ is continuous on an open interval (a,b) if it is continuous at each point $x=c$ in that interval.

A function $f(x)$ is cts on $[a,b]$ if it is cts on (a,b) and

$$f(a)=\lim_{x\to a^+} f(x) \qquad f(b)=\lim_{x\to b^-} f(x)$$





cts on (a,b)
not cts on $[a,b]$

Ex: $f(x) = \dfrac{x+2}{x-3}$

- cts on $(-2, 3)$
- cts on $[-2, 3]$

- $f(x)$ is not cts on $[-2, 3]$ b/c $f(3)$ DNE.

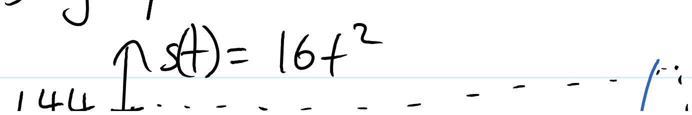
- $f(x)$ is cts on $(-2, 3)$ b/c we can evaluate the function at any value between -2 and 3. The limit is going to be the same as the func. value.

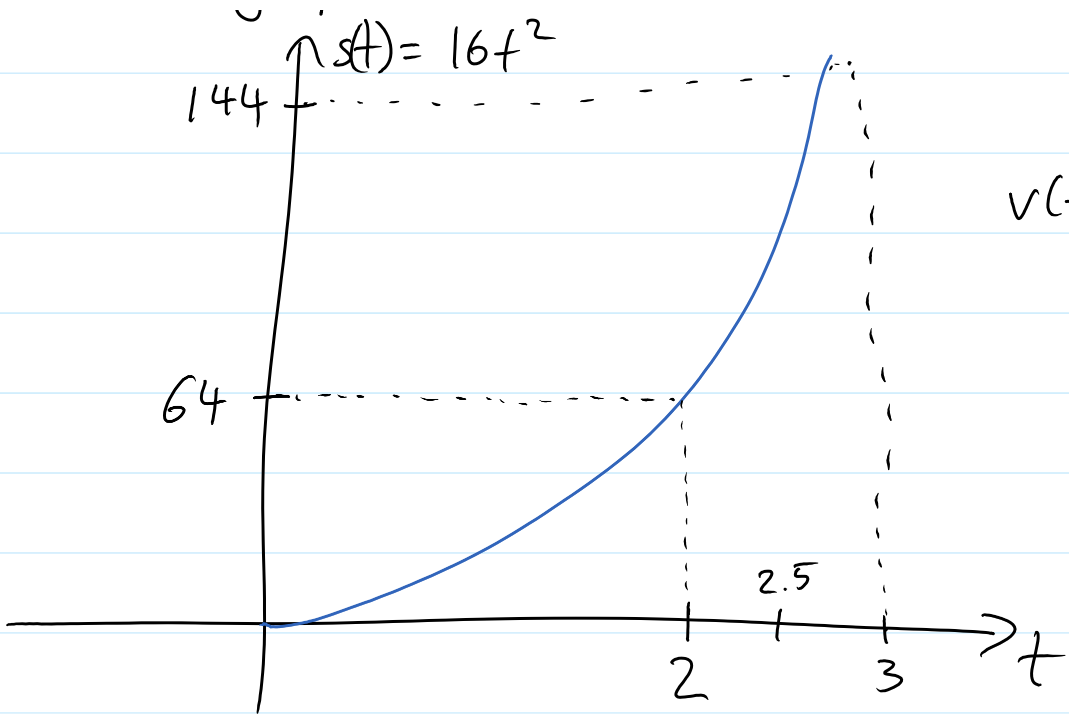
Section 2.1 The derivative

Ex: If air resistance is neglected, an object dropped from a great height will fall $s(t) = 16t^2$ feet in t seconds.

a) What is the objects velocity (speed) after $t=2$ sec?

b) graph $s(t)$.





$v(t)$ .. velocity

average velocity between $t=2, t=3$: $\frac{s(3)-s(2)}{3-2} = \frac{144-64}{1}$

$= 80$ ft/s

ave. vel. $t=2, t=2.5$: $\frac{s(2.5)-s(2)}{2.5-2}$

$h=0.5$

$= \frac{100-64}{0.5} = 72$ ft/s

$t=2, t=2.1$: $\frac{s(2.1)-s(2)}{2.1-2} = \frac{70.56-64}{0.1} = 65.6$ ft/s

$h=0.1$

time difference is h ($h>0$)

$$\frac{s(2+h)-s(2)}{2+h-2} = \frac{16(2+h)^2 - 64}{h}$$

$$= \frac{16(4+4h+h^2) - 64}{h} = \frac{\cancel{64} + 64h + 16h^2 \cancel{-64}}{h}$$

$$= 64 + 16h$$

To find the velocity at $t=2$:

$$\lim_{h\to 0} 64+16h = 64+16\cdot 0 = \boxed{64}\ \text{ft/s}$$