$$\lim_{x \to 10^-} \frac{x^2+100}{x-10} = \frac{200}{0^-} = \boxed{-\infty}$$

$$\frac{100+100}{\cancel{10}-10} = \frac{200}{0^-}$$

9.9
-0.1

Section 2.1

Def: The derivative of the function $f(x)$ with respect to x is the function $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}.$$

If the function $f(x)$ doesn't have a derivative at $x=c$, we say "f is not differentiable at $x=c$".

Ex: find $f'(x)$, $f(x) = 3x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h} = \lim_{h \to 0} \frac{\cancel{h}(6x+3h)}{\cancel{h}}$$

$$= 6x + 3 \cdot 0 = \boxed{6x}$$

Note: • The slope of the tangent line to the

Note: • The slope of the tangent line to the curve $y=f(x)$ at $(c, f(c))$ is $m=f'(c)$.

• The rate of change of $f(x)$ with respect to x when $x=c$ is $f'(c)$.

Ex: Find the tangent line to $y=x^3$ at $x=2$.

$y(2)=2^3=8$ point: $(2,8)$

$$y'(x)=\lim_{h\to 0}\frac{y(x+h)-y(x)}{h}=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2\cancel{h}+3xh^{\cancel{2}}+h^{\cancel{3}\,2}-\cancel{x^3}}{\cancel{h}}$$

$$=3x^2+3x\cdot 0+0^2=3x^2$$

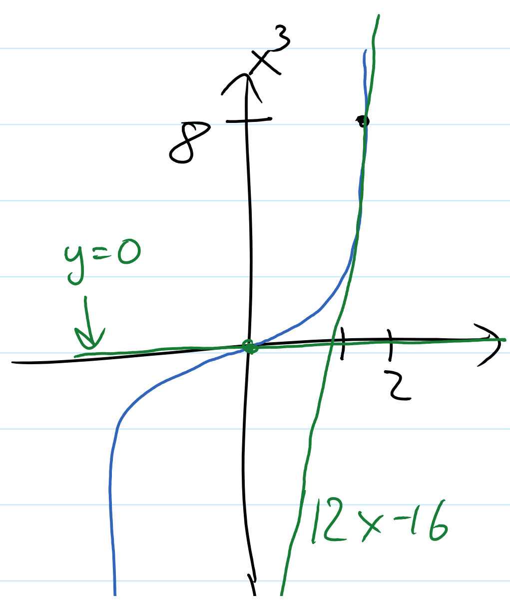
$m=y'(2)=3\cdot 2^2=12$

$\underline{y-y_0=m(x-x_0)}$ | $y=mx+b$

$y-8=12(x-2)$

$y=12x-24+8$

$\boxed{y=12x-16}$

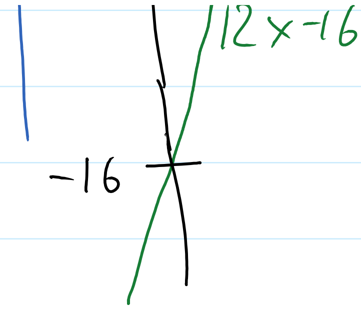


$\boxed{\text{at } x=0}$

$$y'(0)=3\cdot 0^2=0$$

$$y-0=0(x-0)$$

$$\boxed{y=0}$$



<u>Ex:</u> A company has profit

$$P(x)=-400x^2+6800x-12000.$$

x ... in thousands of units

At what rate should we expect profit to be changing with respect to the level of production x when 9000 units are produced? Is the profit increasing or decreasing?

Find $P'(9)$

$$P'(x)=\lim_{h\to 0}\frac{P(x+h)-P(x)}{h}$$

$$=\lim_{h\to 0}\frac{-400(x+h)^2+6800(x+h)-12000-(-400x^2+6800x-12000)}{h}$$

$$=\lim_{h\to 0}\frac{-400(x^2+2xh+h^2)+\cancel{6800x}+6800h-\cancel{12000}+400x^2-\cancel{6800x}+\cancel{12000}}{h}$$

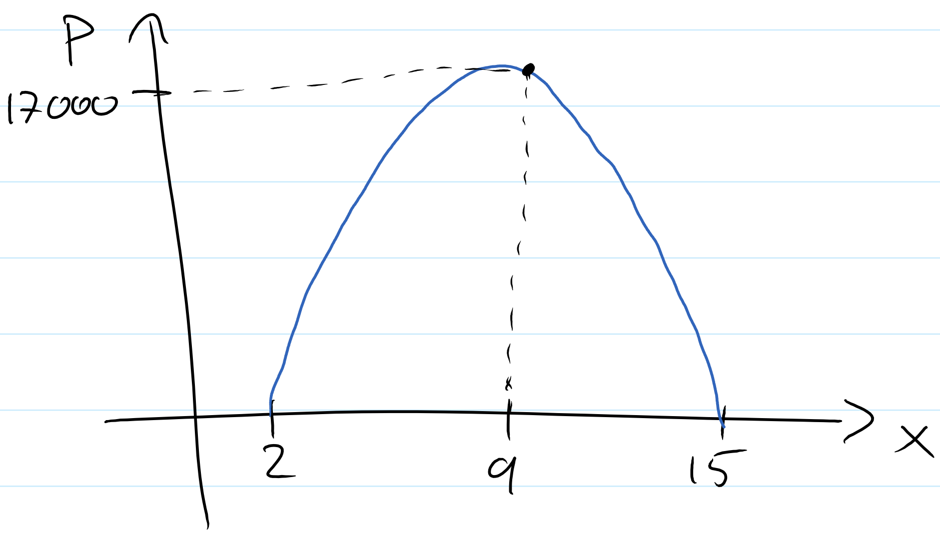
$$=\lim_{h\to 0}\frac{-\cancel{400x^2}-800x\cancel{h}-400h^{\cancel{2}}+6800\cancel{h}+\cancel{400x^2}}{\cancel{h}}$$

$= -800x - 400 \cdot 0 + 6800 = \boxed{-800x + 6800}$

$P'(9) = -800 \cdot 9 + 6800 = -7200 + 6800$
$= \boxed{-400}$

The profit is changing at the rate of
-400 dollars/1000 units

The profit is decreasing.



f'(c) exists

Theorem: If the function f is differentiable at $x = c$, then
- f is increasing at $x = c$, if $f'(c) > 0$
- f is decreasing at $x = c$, if $f'(c) < 0$.