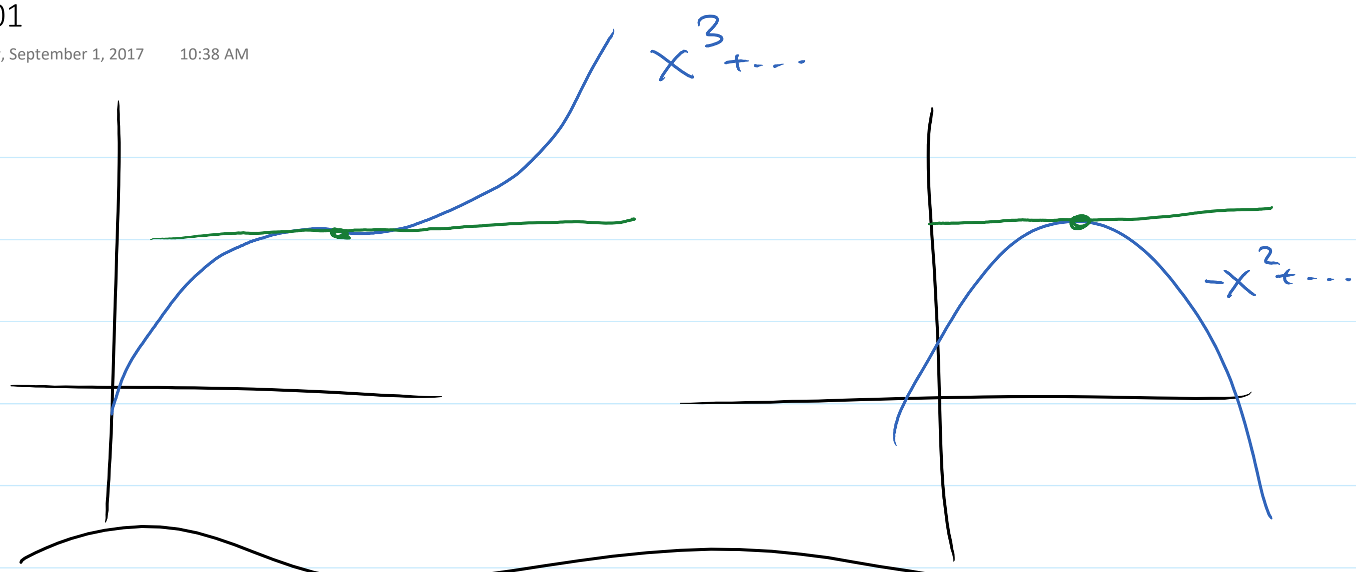


Notation:

let $f(x)$ be differentiable at $x=c$, then

$$f'(c) = \left.\frac{df}{dx}\right|_{x=c} = \left. f'(x)\right|_{x=c}$$

If $y = f(x)$, then

$$f'(x) = y' = \frac{dy}{dx}$$

Ex: $f(x) = \sqrt{x}$, find the equation of the tan. line at $x = 4$.

$(4, 2)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$= \frac{1}{\sqrt{x+0}+\sqrt{x}} = \frac{1}{2\sqrt{x}} \leftarrow \text{derivative}$$

$$m = f'(4) = \frac{1}{2 \cdot 2} = \frac{1}{4}$$

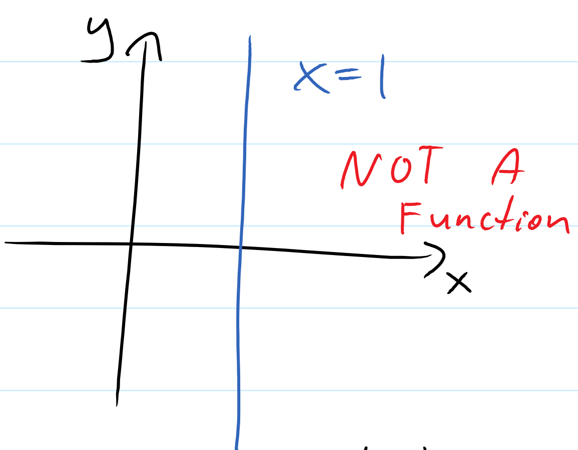
$$y - y_0 = m(x - x_0)$$

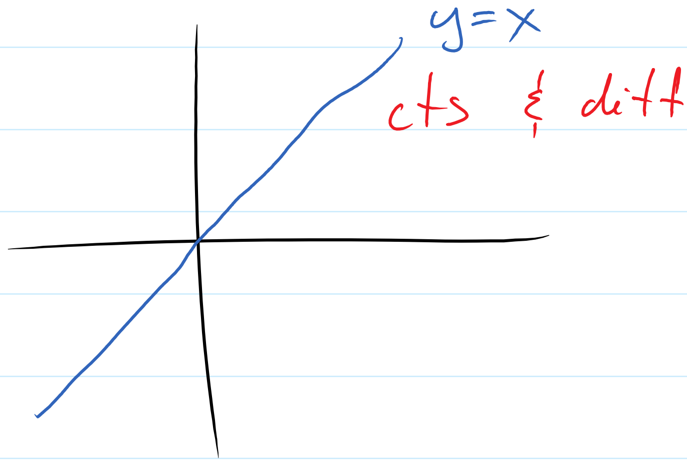
$$y - 2 = \frac{1}{4}(x-4)$$

$$y = \frac{1}{4}x - 1 + 2$$

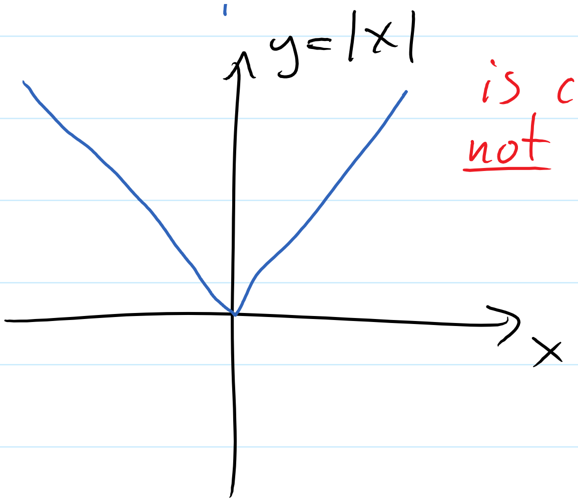
$$\boxed{y = \frac{1}{4}x + 1}$$

Q: Find a function that is continuous, but not differentiable at a point.





$y = |x|$ is cts but



is cts but <u>not</u> diff at $x=0$

<u>Q:</u> Is there a function that is differentiable but not cts?

<u>Thm:</u> If $f(x)$ is differentiable at $x=c$, then $f(x)$ is continuous at $x=c$.

<u>Section 2.2</u>

① Constant func: $y=f(x)=c$

$$y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{c-c}{h} = \lim_{h\to 0} \frac{0}{h}$$

$$= \lim_{h\to 0} 0 = 0$$

$$\boxed{\frac{d}{dx}(c) = (c)' = 0}$$

(2) Polynomials

$f(x) = x$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}} = \lim_{h\to 0} 1 = 1$$

$$\boxed{\frac{d}{dx}(x) = (x)' = 1}$$

$f(x) = x^2$

$$\boxed{\frac{d}{dx}(x^2) = (x^2)' = 2x}$$

$$\boxed{\frac{d}{dx}(x^3) = (x^3)' = 3x^2}$$

$f(x) = x^n$, n .. positive int

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k = \underbrace{\binom{n}{0}}_{=1} a^n + \underbrace{\binom{n}{1}}_{=n} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n} b^n$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{x^n} + n x^{n-1} \cancel{h} + \binom{n}{2} x^{n-2} h^{\cancel{2}\,1} + \dots + h^{\cancel{n}\,n-1} - \cancel{x^n}}{\cancel{h}}$$

$$= \lim_{h\to 0} n x^{n-1} + \binom{n}{2} \cancel{x^{n-2} h} + \binom{n}{3} \cancel{x^{n-3} h^2} + \dots + \cancel{h^{n-1}}$$

$$= \lim_{h \to 0} n x^{n-1} + \binom{n}{2} \cancel{x^{n-2} h} + \binom{n}{3} \cancel{x^{n-3} h^2} + \dots + \cancel{h^{n-1}}$$

$$= n x^{n-1}$$

$$\boxed{\frac{d}{dx}(x^n) = (x^n)' = n x^{n-1},} \text{ where } n \neq 0.$$

$\frac{d}{dx}(c \cdot f(x))$, where c is a constant.

$$(c \cdot f(x))' = \lim_{h \to 0} \frac{c f(x+h) - c f(x)}{h} = \lim_{h \to 0} \frac{c(f(x+h) - f(x))}{h}$$

$$= c \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = c \cdot f'(x)$$

$$\boxed{(c \cdot f(x))' = c \cdot f'(x)}$$

Find: $\frac{d}{dx}(5x^2) = 5 \cdot (x^2)' = 5 \cdot (2 \cdot x^1) = 5 \cdot 2 \cdot x$

$= \boxed{10x}$

$$\frac{d}{dx}(-3x^5) = -3 \cdot (x^5)' = -3 \cdot 5 x^{5-1} = \boxed{-15x^4}$$

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx}\left(f(x)+g(x)\right)=\lim_{h\to 0}\frac{f(x+h)+g(x+h)-f(x)-g(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}$$