09/18

Monday, September 18, 2017 11:57 AM

Section 2.2

differentiation rules:

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}, \quad n \neq 0$$

$$\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}(f(x))$$

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

Ex: $f(x) = 2x^5 - 3x^{-7}$, find $f'(x) = \frac{d}{dx} f(x)$.

$$f'(x) = (2x^5 - 3x^{-7})' = (2x^5)' - (3x^{-7})'$$

$$= \frac{d}{dx}(2x^5) - \frac{d}{dx}(3x^{-7})$$

$$= 2(x^5)' - 3(x^{-7})'$$

$$= 2 \cdot 5 x^{5-1} - 3 \cdot (-7) x^{-7-1} = \boxed{10x^4 + 21x^{-8}}$$

$$= \boxed{10x^4 + \frac{21}{x^8}}$$

Ex: $f(x) = \underbrace{x^{-2}(x^3 + 31)}$, $f'(x) = ?$

Ex: $f(x) = \underbrace{x^{-2}(x+31)}_{\text{Product of two functions!}}$, $f'(x) = ?$

$$f(x) = x^{-2+3} + 31x^{-2} = x + 31x^{-2}$$

$$f'(x) = (x + 31x^{-2})' = 1\cdot x^0 + 31(x^{-2})' = 1 + 31\cdot(-2)x^{-3}$$
$$= \boxed{1 - 62x^{-3}}$$

Ex: $y = 5x^3 - 4x^2 + 12x - 8$, find $y' = \frac{dy}{dx}$

$$y' = 5(x^3)' - 4(x^2)' + 12(x)' - (8)'$$
$$= 5\cdot 3x^2 - 4\cdot 2x^1 + 12\cdot 1 - 0 = \boxed{15x^2 - 8x + 12}$$

$$y = 4x^{10} + 3x^4 - 5x^2 - 3x + 7$$
$$y' = 4\cdot 10x^9 + 3\cdot 4x^3 - 5\cdot 2x - 3 + 0$$
$$= \boxed{40x^9 + 12x^3 - 10x - 3}$$

Relative and Percentage Rates of Change.

The relative rate of change of a quantity $Q(x)$ with respect to x is given by the ratio:

$$\frac{Q'(x)}{Q(x)}$$

The percentage rate of change of $Q(x)$ with respect to x is

$$\frac{100\, Q'(x)}{Q(x)}$$

Ex: Suppose that the GDP of a country was $N(t) = t^2 + 5t + 106$ billion dollars t years after 2000.

a) At what rate was the GDP changing with respect to time in 2010?

$$N'(10) = ?$$

$$N'(t) = 2t^1 + 5 + 0 = 2t + 5$$

$$N'(10) = 2 \cdot 10 + 5 = \boxed{25} \ \frac{\text{bill. of dollars}}{\text{year}}$$

b) At what percentage rate of change was the GDP changing in 2010?

$$\frac{100 \cdot N'(10)}{N(10)} = \frac{100 \cdot 25}{10^2 + 5 \cdot 10 + 106} = \frac{100 \cdot 25}{256} = 9.77$$

$$\boxed{9.77\%}$$

Note: If the position at time t of an object moving along a straight line is given by $s(t)$, then the object has is given by $s(t)$, then the object has

velocity: $v(t) = s'(t) = \frac{d}{dt}(s(t))$

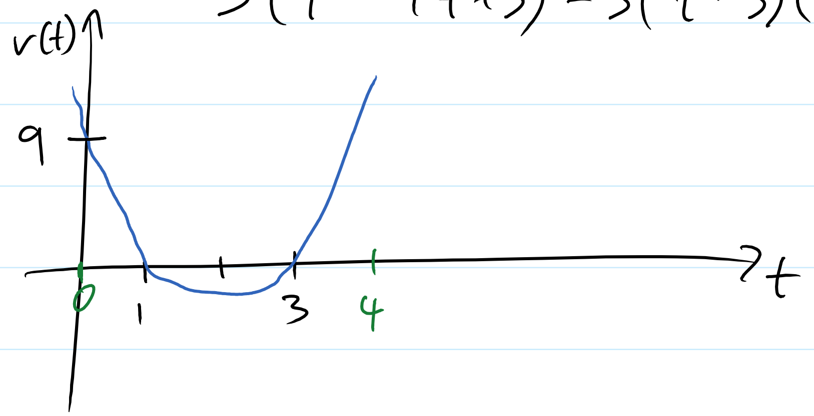
and

acceleration: $a(t) = v'(t) = \frac{d}{dt}(v(t))$

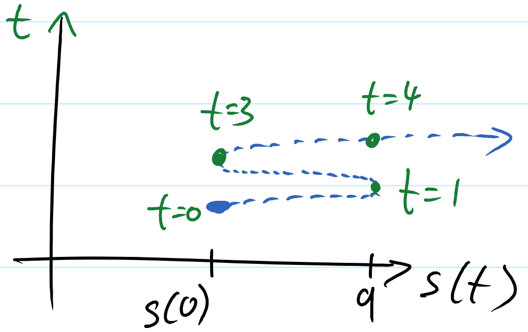
Ex: the position of an object is given by $s(t) = t^3 - 6t^2 + 9t + 5$

a) Find the velocity between $t=0$, $t=4$.

$$v(t) = s'(t) = (t^3 - 6t^2 + 9t + 5)' = 3t^2 - 6\cdot 2\cdot t + 9 + 0$$
$$= \boxed{3t^2 - 12t + 9}$$
$$= 3(t^2 - 4t + 3) = 3(t-3)(t-1)$$



b) Find the total distance traveled between $t=0$, $t=4$.



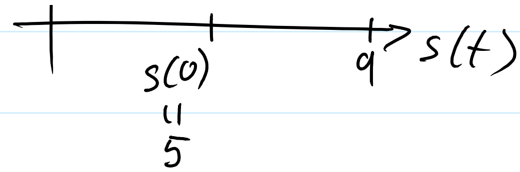
$s(1) = 1^3 - 6\cdot 1^2 + 9\cdot 1 + 5 = 9$

$s(3) = 5$

$s(4) = 9$

$s(3) = 5$

$s(4) = 9$



$s(t) = t^3 - 6t^2 + 9t + 5$

Total distance: $|s(1) - s(0)| + |s(3) - s(1)| + |s(4) - s(3)|$

$= |9 - 5| + |5 - 9| + |9 - 5|$

$= \boxed{12}$