9/20

Wednesday, September 20, 2017 11:53 AM

Section 2.3

$$\begin{cases} f(x) = (x+3)(x^2-2) = x^3-2x+3x^2-6 = x^3+3x^2-2x-6 \\ f'(x) = 3x^2+6x-2 \end{cases}$$

$$f(x) = (x+3)(x^2-2)^{31}$$

The Product Rule

$$(f(x)g(x))' = f'(x)g(x) + g'(x)f(x)$$
$$= g(x)f'(x) + f(x)g'(x)$$

The Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{g^2(x)}$$

$$= \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)} \qquad \left[\frac{hi}{lo}\right]' = \frac{lo\,Dhi - hi\,Dlo}{(lo)^2}$$

Ex: $f(x) = \underbrace{(x+3)}_{f}\underbrace{(x^2-2)}_{g}$, Find $f'(x)$

$$f'(x) = (x+3)'(x^2-2) + (x^2-2)'(x+3)$$
$$= (1+0)(x^2-2) + (2x-0)(x+3)$$
$$= x^2-2+2x^2+6x = \boxed{3x^2+6x-2}$$

Ex: $y = (2x+1)(2x^2-x-1)$

1) Find y':

$$y' = (2x+1)'(2x^2-x-1) + (2x^2-x-1)'(2x+1)$$
$$= (2+0)(2x^2-x-1) + (4x-1-0)(2x+1)$$
$$= 4x^2-2x-2 + (4x-1)(2x+1)$$
$$= 4x^2-2x-2 + 8x^2+2x-1 = \boxed{12x^2-3}$$

2) Find the tangent line at $x=1$.

$m = y'(1) = 12\cdot 1 - 3 = 9$

y-coordinate: $y(1) = (2\cdot 1+1)(2\cdot 1^2-1-1) = 3\cdot 0 = 0$

point: $(1,0)$ slope $= 9$

$$y - 0 = 9(x-1)$$
$$\boxed{y = 9x-9}$$

3) Find all the points where the tangent line is

3) Find all the points where the tangent line is horizontal

$$12x^2 - 3 = 0$$
$$3(4x^2 - 1) = 0$$
$$4x^2 - 1 = 0$$
$$x^2 = 1/4$$
$$x = \pm\sqrt{1/4} = \boxed{\pm \frac{1}{2}}$$

Ex: Differentiate: $y = \dfrac{x^2 - 5x + 7}{2x}$

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

$$y' = \frac{(x^2-5x+7)'(2x) - (2x)'(x^2-5x+7)}{(2x)^2}$$

$$= \frac{(2x-5)(2x) - 2(x^2-5x+7)}{4x^2} = \frac{4x^2 - 10x - 2x^2 + 10x - 14}{4x^2}$$

$$= \frac{2x^2 - 14}{4x^2} = \boxed{\frac{x^2 - 7}{2x^2}} = \frac{x^2}{2x^2} - \frac{7}{2x^2} = \frac{1}{2} - \frac{7}{2}x^{-2}$$

Also:

$$y = \frac{x^2 - 5x + 7}{2x} = \frac{x^2}{2x} - \frac{5x}{2x} + \frac{7}{2x} = \frac{1}{2}x - \frac{5}{2} + \frac{7}{2}x^{-1}$$

$$y' = \frac{1}{2} - 0 + \frac{7}{2}\cdot(-1)x^{-2} = \boxed{\frac{1}{2} - \frac{7}{2}x^{-2}}$$

Ex: A population of bacteria in a culture is P million where

$$P(t) = \frac{t+1}{t^2+t+4}$$

1) At what rate is the population changing with respect to time when $t=0$?

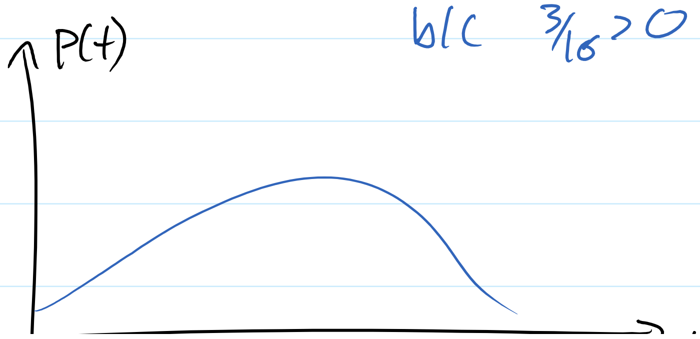
$$P'(t) = \frac{(t+1)'(t^2+t+4)-(t^2+t+4)'(t+1)}{(t^2+t+4)^2}$$

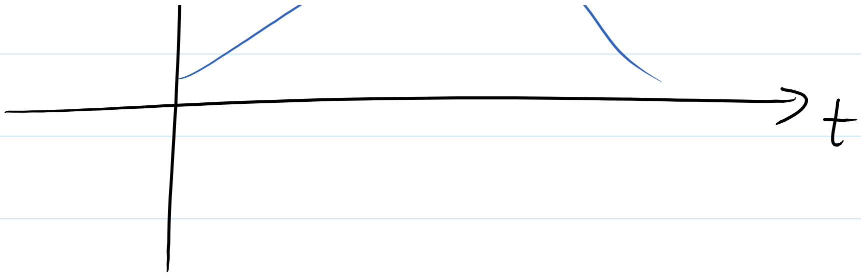
$$= \frac{(1+0)(t^2+t+4)-(2t+1+0)(t+1)}{(t^2+t+4)^2}$$

$$= \frac{t^2+t+4-(2t^2+t+2t+1)}{(t^2+t+4)^2}$$

$$= \frac{t^2+t+4-2t^2-3t-1}{(t^2+t+4)^2} = \frac{-t^2-2t+3}{(t^2+t+4)^2}$$

$$P'(0) = \frac{3}{4^2} = \boxed{\frac{3}{16}}$$ → the population increases at $t=0$. b/c $3/16 > 0$





2) Find when does the population reach max.

$P'(t) = 0 \Longleftrightarrow$ tangent line is horizontal.

$$\frac{-t^2-2t+3}{(t^2+t+4)^2} = 0$$

$$-t^2-2t+3=0$$

$$t^2+2t-3=0$$

$$(t-1)(t+3)=0$$

$$\boxed{t=1}, \quad t=-3$$

Ex: $y = \frac{2}{3x^2} - \frac{x}{3} + \frac{4}{5} + \frac{x+1}{x} = \frac{3}{2}\cdot x^{-2} - \frac{1}{3}x + \frac{4}{5} + 1 + x^{-1}$

$+\frac{x}{x} + \frac{1}{x}$

$\frac{2}{3}\cdot\frac{1}{x^2} + \frac{1}{3}\cdot\frac{x}{1}$