9/22

Friday, September 22, 2017 9:40 AM

Exam on Wednesday, 9/27 in class.
↳ Online office hour on Tuesday, 9/26, $\boxed{\text{6:45-7:45 PM}}$
Please send me an email if you want to see a specific problem solved.

Section 2.3

Ex: find $\frac{dy}{dx}$, where $y = \frac{2}{3x^2} - \frac{x}{3} + \frac{4}{5} + \frac{x+1}{x}$

$$y = \frac{2}{3}x^{-2} - \frac{1}{3}x + \frac{4}{5} + \underbrace{1 + x^{-1}}_{\frac{x}{x} + \frac{1}{x}}$$

$$\frac{dy}{dx} = \frac{2}{3}\cdot(-2)x^{-3} - \frac{1}{3}\cdot 1 + 0 + 0 + (-1)x^{-2}$$

$$= \boxed{-\frac{4}{3}x^{-3} - \frac{1}{3} - x^{-2}} = \boxed{\frac{-4}{3x^3} - \frac{1}{3} - \frac{1}{x^2}}$$

Find the second derivative of the previous func

↳ $y'' = \frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = \left(-\frac{4}{3}x^{-3} - \frac{1}{3} - x^{-2}\right)' = -\frac{4}{3}\cdot(-3)x^{-4} - 0 - (-2)x^{-3}$$

$$\frac{d^2y}{dx^2} = \left(-\frac{4}{3}x^{-3} - \frac{1}{3} - x^{-2}\right)' = -\frac{4}{3}\cdot(-3)x^{-4} - 0 - (-2)x^{-3}$$
$$= \boxed{4x^{-4} + 2x^{-3}}$$

Find $f''(x)$, where $f(x) = x^2(3x+1) = 3x^3 + x^2$

$$f'(x) = 9x^2 + 2x$$
$$f''(x) = \boxed{18x + 2}$$

Notation: the high order derivatives: n^{th} derivative of $f(x)$

$$f^{(n)}(x) = \frac{d^n f}{dx^n}$$

Find the 4^{th} derivative of $\frac{1}{x}$.

$$y = \frac{1}{x} = x^{-1}$$

$$y' = -x^{-2}$$
$$y'' = -(-2)x^{-3} = 2x^{-3}$$
$$y^{(3)} = -6x^{-4}$$
$$y^{(4)} = -6\cdot(-4)x^{-5} = \boxed{24x^{-5}}$$
$$\vdots$$
$$y^{(10)} = (10!)\,x^{-11} = 2\cdot3\cdot4\cdot\ldots\cdot8\cdot9\cdot10\,x^{-11}$$

Section 2.4 – The Chain Rule

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Diff: $y = (2x-3)^2 = 4x^2 - 12x + 9$

$$y' = \boxed{8x - 12}$$

Think of $y(x) = (2x-3)^2 = y(u(x)) = u^2$

$u = 2x-3$

The Chain Rule

Given two differentiable functions, $f(x), g(x)$, the derivative of $(f(g(x)))'$ is given by

$$\frac{d}{dx}(f(g(x))) = \left[\frac{d}{du} f(u)\right]\left[\frac{d}{dx} g(x)\right]$$

$$\frac{d}{dx}\left(y(u(x))\right) = \left[\frac{d}{du} y(u)\right]\left[\frac{d}{dx} u(x)\right]$$

$$= y'(u) \cdot u'(x)$$

$y(x) = (2x-3)^2 \begin{cases} y(u) = u^2 \\ u(x) = 2x-3 \end{cases}$

$$\frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{du}{du} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d}{du}(u^2) \cdot \frac{d}{dx}(2x-3) = 2u \cdot (2) = 4u$$
$$= 4(2x-3) = \boxed{8x-12}$$

Find $\frac{dy}{dx}$, $y = \sqrt{3x-x^2}$ $\begin{cases} y = u^{1/2} \\ u = 3x - x^2 \end{cases}$

$$\frac{dy}{dx} = (u^{1/2})' \cdot (3x-x^2)' = \frac{1}{2} u^{-1/2} \cdot (3-2x)$$
$$= \frac{1}{2\sqrt{3x-x^2}}(3-2x) = \boxed{\frac{3-2x}{2\sqrt{3x-x^2}}}$$

Differenciate: $f(x) = (x^2+2)^3 - 3(x^2+2)^2 + 1$

$$f'(x) = \Big(\underbrace{(x^2+2)^3}_{\substack{u^3 \\ u = x^2+2}}\Big)' - 3\Big(\underbrace{(x^2+2)^2}_{\substack{u^2 \\ u = x^2+2}}\Big)' + 0$$
$$= 3(x^2+2)^2 \cdot (x^2+2)' - 3 \cdot 2(x^2+2) \cdot (x^2+2)'$$
$$= 3(x^2+2)^2 \cdot 2x \quad - 6(x^2+2) \cdot 2x$$
$$= \boxed{6x(x^2+2)^2 - 12x(x^2+2)}$$

The General Power Rule:

$$\frac{d}{dx}(f(x))^n = n \cdot (f(x))^{n-1} \cdot f'(x)$$

$y = \sqrt{x^2+3x+2}$ Find y'

$= (x^2+3x+2)^{1/2}$

$y' = \frac{1}{2}(x^2+3x+2)^{-1/2} \cdot (x^2+3x+2)'$

$= \frac{1}{2\sqrt{x^2+3x+2}} \cdot (2x+3) = \boxed{\frac{2x+3}{2\sqrt{x^2+3x+2}}}$

Differenciate: $f(x) = \frac{1}{(2x+3)^5} = (2x+3)^{-5}$

$f'(x) = -5(2x+3)^{-6} \cdot (2x+3)' = -5(2x+3)^{-6} \cdot 2$

$= \boxed{-10(2x+3)^{-6}}$

$y = (3x+1)^4 (2x-1)^5$, Find y'

$y' = (2x-1)^5 \left[(3x+1)^4\right]' + (3x+1)^4 \left[(2x-1)^5\right]'$

$= (2x-1)^5 \cdot 4(3x+1)^3 \cdot (3x+1)' + (3x+1)^4 \cdot 5(2x-1)^4 \cdot (2x-1)'$

$= (2x-1)^5 \cdot 4(3x+1)^3 \cdot 3 + (3x+1)^4 \cdot 5(2x-1)^4 \cdot 2$

$= 2(2x-1)^4(3x+1)^3\left[6(2x-1) + 5(3x+1)\right]$