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Section 2.4

Differenciale: $f(x) = \dfrac{(x+2)^5}{(2-x)^4}$ $\quad \left(\dfrac{f}{g}\right)' = \dfrac{gf' - fg'}{g^2}$

$$f'(x) = \frac{(2-x)^4 \cdot [(x+2)^5]' - (x+2)^5 \cdot [(2-x)^4]'}{[(2-x)^4]^2}$$

$$= \frac{(2-x)^4 5\cdot(x+2)^4\cdot(x+2)' - (x+2)^5 \cdot 4(2-x)^3 \cdot (2-x)'}{(2-x)^8}$$

$$= \frac{(2-x)^4 \cdot 5(x+2)^4 \cdot 1 - (x+2)^5 4(2-x)^3(-1)}{(2-x)^8}$$

$$= \frac{5(2-x)^4(x+2)^4 + 4(x+2)^5(2-x)^3}{(2-x)^8}$$

$$= \frac{(2-x)^3(x+2)^4[5(2-x)+4(x+2)]}{(2-x)^{8\,5}} = \frac{(x+2)^4(10-5x+4x+8)}{(2-x)^5}$$

$$= \boxed{\frac{(x+2)^4(18-x)}{(2-x)^5}}$$

Find tangent line to $f(x) = \left(\dfrac{x+9}{x-9}\right)^3$ at $x = 6$.

$$f'(x) = 3\left(\frac{x+9}{x-9}\right)^2 \cdot \left(\frac{x+9}{x-9}\right)'$$

$$\left(\frac{x+9}{x-9}\right)' = \frac{(x-9)(x+9)' - (x+9)(x-9)'}{} = \frac{x-9-x-9}{} = \frac{-18}{}$$

$$\left(\frac{x+9}{x-9}\right)' = \frac{(x-9)(x+9)' - (x+9)(x-9)'}{(x-9)^2} = \frac{x-9-x-9}{(x-9)^2} = \frac{-18}{(x-9)^2}$$

$$f'(x) = 3\left(\frac{x+9}{x-9}\right)^2 \cdot \frac{-18}{(x-9)^2} = \frac{3}{1} \cdot \frac{(x+9)^2}{(x-9)^2} \cdot \frac{-18}{(x-9)^2} = \frac{-54(x+9)^2}{(x-9)^4}$$

$$f'(6) = \frac{-54(15)^2}{(-3)^4} = -150$$

$$f(6) = \left(\frac{15}{-3}\right)^3 = (-5)^3 = -125$$

$$y-(-125) = -150(x-6)$$
$$y+125 = -150x+900$$
$$\boxed{y = -150x+775}$$

Note:

$$\frac{x-c}{x+c} = \frac{x-c+c-c}{x+c} = \frac{x+c-2c}{x+c} = \frac{x+c}{x+c} - \frac{2c}{x+c}$$

$$= 1 - 2c(x+c)^{-1}$$

$$\frac{x-9}{x+9} = \frac{x+9-18}{x+9} = \underbrace{1-18(x+9)^{-1}}$$

$$\frac{d}{dx}(\cdots) = 0 - 18(-1)(x+9)^{-2} \cdot 1$$
$$= 18(x+9)^{-2} = \frac{18}{(x+9)^2}$$

Diff: $y = 3(5x+1)^5(6x-5)^2$ $\quad (f\cdot g)' = gf' + fg'$

$$y' = 3\left[(6x-5)^2\left[(5x+1)^5\right]' + (5x+1)^5\left[(6x-5)^2\right]'\right]$$

$$= 3(6x-5)^2 5(5x+1)^4 \cdot 5 + 3(5x+1)^5 2(6x-5)\cdot 6$$

$$= 3(6x-5)^2 5(5x+1)^4 \cdot 5 + 3(5x+1)^5 2(6x-5)\cdot 6$$
$$= 75(6x-5)^2(5x+1)^4 + 36(5x+1)^5(6x-5)$$
$$= 3(6x-5)(5x+1)^4\left[25(6x-5)+12(5x+1)\right]$$
$$= 3(6x-5)(5x+1)^4(150x-125+60x+12)$$
$$= \boxed{3(6x-5)(5x+1)^4(210x-113)}$$

Find the second derivative of:

$$y = \frac{(2y-4)^4}{(2y-1)^4} = \left(\frac{2y-4}{2y-1}\right)^4 = \left(\frac{2y-1}{2y-1} - \frac{3}{2y-1}\right)^4$$
$$= \left(1-3(2y-1)^{-1}\right)^4$$

$$y' = 4\left(1-3(2y-1)^{-1}\right)^3 \cdot \left(0-3\left[(2y-1)^{-1}\right]'\right)$$
$$= 4\left(1-3(2y-1)^{-1}\right)^3 \cdot (-3)(-1)(2y-1)^{-2}\cdot 2$$
$$= 24\left(1-3(2y-1)^{-1}\right)^3 \cdot (2y-1)^{-2} \quad \text{(i)}$$

$$= 24 \cdot \frac{(2y-4)^3}{(2y-1)^5} \quad \text{(ii)}$$

$$y'' = 24\,\frac{(2y-1)^5 3(2y-4)^2\cdot 2 - (2y-4)^3\cdot 5(2y-1)^4\cdot 2}{\left((2y-1)^5\right)^2}$$

$$= 24 \cdot \frac{6(2y-1)^5(2y-4)^2 - 10(2y-4)^3(2y-1)^4}{(2y-1)^{10}}$$

$$= 24\,\frac{2(2y-1)^4(2y-4)^2\left[3(2y-1)-5(2y-4)\right]}{(2y-1)^{10}}$$

$$= 24\,\frac{2(2y-1)(2y-4)\,[5(2y-1)-5(2y-4)]}{(2y-1)^{10\,6}}$$

$$= 24\,\frac{2(2y-4)^2(6y-3-10y+20)}{(2y-1)^6} = \boxed{\frac{48(2y-4)^2(17-4y)}{(2y-1)^6}}$$

$$\lim_{x\to 2}\frac{4x(x-2)}{\sqrt{x^3-3x}} = \boxed{0} \qquad \frac{4\cdot 2(2-2)}{\sqrt{8-6}} = \frac{8\cdot 0}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$$

$$\lim_{x\to 2}\frac{\sqrt{x^3-3x}}{4x(x-2)} \quad \boxed{\text{DNE}} \qquad \frac{\sqrt{2}}{0}$$

- $\lim_{x\to 2^+}\frac{\sqrt{x^3-3x}}{4x(x-2)}$ $\qquad x = 2.1$ $\qquad \frac{\sqrt{2}}{8\cdot(2.1-2)} = \frac{\sqrt{2}}{8(0.1)} = \frac{\sqrt{2}}{0^+} = +\infty$

- $\lim_{x\to 2^-}\frac{\sqrt{x^3-3x}}{4x(x-2)}$ $\qquad x = 1.9$ $\qquad \frac{\sqrt{2}}{8\cdot(-0.1)} = \frac{\sqrt{2}}{-8\cdot 0.1} = \frac{\sqrt{2}}{0^-} = -\infty$

Find y'', where $y = (1-6x^6)^7$

$$y' = 7(1-6x^6)^6\cdot(-6\cdot 6x^5)$$
$$= -252\,x^5(1-6x^6)^6$$

$$y'' = -252 \cdot \left[(1-6x^6)^6 \cdot (x^5)' + x^5 \left[(1-6x^6)^6 \right]' \right]$$

$$= -252 \left[5x^4 (1-6x^6)^6 + x^5 \cdot 6 (1-6x^6)^5 \cdot (-6 \cdot 6x^5) \right]$$

$$= -252 \left[5x^4 (1-6x^6)^6 - 216x^{10} (1-6x^6)^5 \right]$$

$$= -252 x^4 (1-6x^6)^5 \left[5(1-6x^6) - 216x^6 \right]$$

$$= \boxed{-252 x^4 (1-6x^6)^5 (5 - 246x^6)}$$

$$= \boxed{252 x^4 (1-6x^6)^5 (246x^6 - 5)}$$