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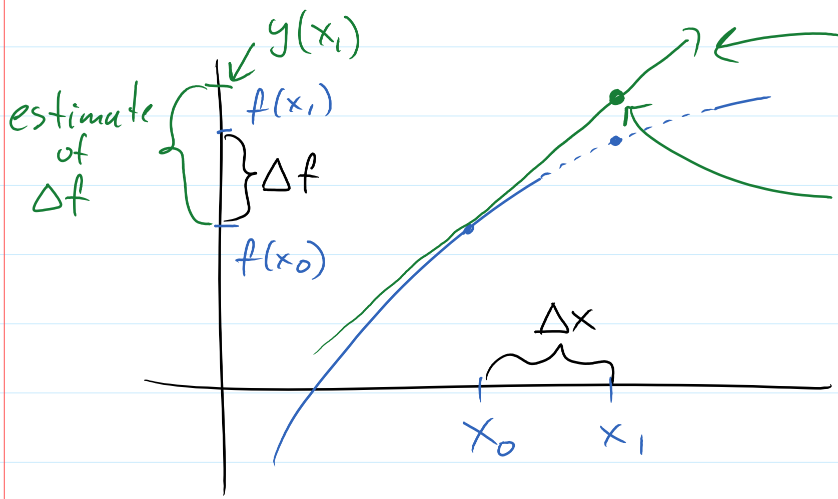

$y - f(x_0) = f'(x_0)(x - x_0)$

$y = f'(x_0)(x - x_0) + f(x_0)$

$y = f'(x_0)(x_1 - x_0) + f(x_0)$

$y = f'(x_0)\Delta x + f(x_0)$

$\Delta x = x_1 - x_0$

$\Delta f = f(x_1) - f(x_0)$

est. of Δf is $g(x_1) - f(x_0)$

$f'(x_0)\Delta x + \cancel{f(x_0)} - \cancel{f(x_0)}$

$\boxed{f'(x_0)\Delta x}$

$$\Delta f \approx f'(x_0)\Delta x$$

- If $x_1 = x_0 + 1 \Rightarrow \Delta x = 1$

$$\Delta f = f(x_1) - f(x_0) = f(x_0+1) - f(x_0) \approx f'(x_0)\cdot 1$$

Section 2.5

Def: If $C(x)$ is the total cost of producing x units of a commodity, then the marginal cost of producing x_0 units is the derivative, $C'(x_0)$.

For x_0 sufficiently large, the marginal cost, $C'(x_0)$, can be used to estimate the additional cost $C(x_0+1) - C(x_0)$ incurred cost, $C'(x_0)$, can be used to estimate the additional cost $C(x_0+1) - C(x_0)$ incurred when the level of production is increased from x_0 to x_0+1.

Similarly, Marginal Profit and Revenue...

EXAMPLE 2.5.1 Studying Marginal Cost and Marginal Revenue

A manufacturer estimates that when *x* units of a particular commodity are produced, the total cost will be $C(x) = \frac{1}{8}x^2 + 3x + 98$ dollars, and furthermore, that all *x* units will be sold when the price is $p(x) = \frac{1}{3}(75 - x)$ dollars per unit.

a. Find the marginal cost and the marginal revenue.

b. Use marginal cost to estimate the cost of producing the 37th unit. What is the actual cost of producing the 37th unit?

c. Use marginal revenue to estimate the revenue derived from the sale of the 37th unit. What is the actual revenue derived from the sale of the 37th unit?

$$C(x) = \frac{1}{8}x^2 + 3x + 98$$
$$R(x) = x\,p(x) = x \cdot \frac{1}{3}(75 - x) = \frac{1}{3}(75x - x^2)$$

a) $\boxed{C'(x) = \frac{1}{4}x + 3}$ $\quad R'(x) = \boxed{\frac{1}{3}(75 - 2x)}$

estimate

b) $C(37) - C(36) \approx C'(36) = \frac{1}{4} \cdot 36 + 3 = 9 + 3 = \boxed{\$12}$

$C(37) - C(36) = 380.125 - 368 = \boxed{\$12.13}$

actual

estimate

c) $R(37) - R(36) \approx R'(36) = \frac{1}{3}(75 - 72) = \boxed{\$1}$

$R(37) - R(36) = 468.67 - 468 = \boxed{\$0.67}$ ← actual

EXAMPLE 2.5.3 Estimating Change in Cost Using a Derivative

Suppose the total cost of manufacturing q hundred units of a certain commodity is C thousand dollars where $C(q) = 3q^2 + 5q + 10$. If the current level of production is 4,000 units, estimate how the total cost will change if 4,050 units are produced.

$C(q) = 3q^2 + 5q + 10$

$4000 \to q = 40$

$4050 \to q = 40.5$

$C'(q) = 6q + 5$

$\Delta q = 0.5$

$C(40.5) - C(40)$

$= \Delta C \approx C'(x_0)\,\Delta q$

$\Delta C \approx C'(40) \cdot 0.5 = (6 \cdot 40 + 5)\frac{1}{2} = \frac{245}{2} = 122.5$

$\boxed{\$122500}$

Def: The differential of x is $dx = \Delta x$, and $y = f(x)$, the differential of y is

$$dy = f'(x)\,dx$$

Ex: Find dy:

- $y = f(x) = x^3 - 7x^2 + 2$

$$\boxed{dy = (3x^2 - 14x)\,dx}$$

- $y = f(x) = (x^2 + 5)(3 - x - 2x^2)$

$$\boxed{dy = \left[(3 - x - 2x^2)(2x) + (x^2 + 5)(-1 - 4x)\right] dx}$$