MAC 2233, Fall 2017

Exam #1

September 27, 2017

Name Key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No calculators are allowed!

Honor Code: On my honor, I have neither received nor given any aid during this examination.

Signature: ______________________

1. (5 points each) Evaluate the following limits algebraically, if they exist:

a) $\lim_{x\to-\infty} \frac{x^2+3x-1}{x^3-4x+2} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to-\infty} \frac{\frac{1}{x}+\frac{3}{x^2}-\frac{1}{x^3}}{1-\frac{4}{x^2}+\frac{2}{x^3}} = \frac{0+0-0}{1-0+0} = \boxed{0}$

b) $\lim_{x\to4} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to4} \frac{\cancel{x-4}}{(\cancel{x-4})(\sqrt{x}+2)} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

c) $\lim_{x\to2} \frac{x-2}{x^2-3x+2} = \lim_{x\to2} \frac{(\cancel{x-2})}{(\cancel{x-2})(x-1)} = \frac{1}{2-1} = \boxed{1}$

d) $\lim_{x\to 2} \frac{x^2+5x}{(x-2)^2(2x+3)}$ $= \frac{4+10}{(0)^2(4+3)} = \frac{14}{0}$ need to check one-sided limits.

$\lim_{x\to 2^-} \frac{x^2+5x}{(x-2)^2(2x+3)} = \frac{14}{(0^-)^2\cdot 7} = \frac{14}{0^+\cdot 7} = \frac{14}{0^+} = \boxed{+\infty}$

$\lim_{x\to 2^+} \frac{x^2+5x}{(x-2)^2(2x+3)} = \frac{14}{(0^+)^2\cdot 7} = \frac{14}{0^+} = \boxed{+\infty}$

The same so

$\lim_{x\to 2} \frac{x^2+5x}{(x-2)^2(2x+3)} = \boxed{\infty}$

2. (5 points) Find the derivative of the function using the **definition of derivative**.

$$f(x) = \frac{2}{x}$$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h\to 0} \frac{\frac{2x-2(x+h)}{x(x+h)}}{h}$

$= \lim_{h\to 0} \frac{\cancel{2x}-\cancel{2x}-2h}{hx(x+h)} = \lim_{h\to 0} \frac{-2}{x(x+h)} = \frac{-2}{x(x+0)} = \boxed{\frac{-2}{x^2}}$

3. (5 points each) Differentiate the following function and simplify the derivative

(a) $f(x) = x^2 - \sqrt{x}$ $= x^2 - x^{1/2}$

$f'(x) = \boxed{2x - \frac{1}{2}x^{-1/2}} = \boxed{2x - \frac{1}{2\sqrt{x}}}$

(b) $f(t) = \frac{2t}{\sqrt{t}} = 2t^{1} \cdot t^{-1/2} = 2\,t^{1/2}$

$$f'(t) = 2 \cdot \tfrac{1}{2} t^{-1/2} = \boxed{t^{-1/2}} = \boxed{\frac{1}{\sqrt{t}}}$$

(c) $f(u) = \frac{1-2u}{1+2u}$

$$f'(u) = \frac{(1+2u)(-2) - (1-2u)\cdot 2}{(1+2u)^2} = \frac{-2 - \cancel{4u} - 2 + \cancel{4u}}{(1+2u)^2}$$

$$= \boxed{\frac{-4}{(1+2u)^2}}$$

4. (10 points each) Find the first and second derivative of the function and simplify your answer

(a) $f(x) = x(3x+4)^5$

$$f'(x) = (3x+4)^5 \cdot 1 + x \cdot 5(3x+4)^4 \cdot 3 = (3x+4)^4[3x+4+15x]$$

$$= \boxed{(3x+4)^4(18x+4)} = \boxed{2(3x+4)^4(9x+2)}$$

$$f''(x) = 2[(9x+2)\,4(3x+4)^3 \cdot 3 + (3x+4)^4 \cdot 9]$$

$$= 2 \cdot 3(3x+4)^3[(9x+2)4 + (3x+4) \cdot 3]$$

$$= 6(3x+4)^3(36x+8+9x+12) = 6(3x+4)^3(45x+20)$$

$$= \boxed{30(3x+4)^3(9x+4)}$$

(b) $s(t) = \frac{4}{3-t}$ $= 4(3-t)^{-1}$

$s'(t) = 4(-1)(3-t)^{-2}\cdot(-1) = \boxed{4(3-t)^{-2}} = \boxed{\frac{4}{(3-t)^2}}$

$s''(t) = 4(-2)(3-t)^{-3}\cdot(-1)$
$= \boxed{8(3-t)^{-3}} = \boxed{\frac{8}{(3-t)^3}}$

5. (10 points) The distance a particle travels in a particle accelerator in CERN is given by the following function

$$s(t) = \frac{32}{t} + 5t^2$$ $= 32t^{-1} + 5t^2$

(a) What is the velocity of the particle when $t = 2$?

$v(t) = s'(t) = -32t^{-2} + 10t = \frac{-32}{t^2} + 10t$

$v(2) = -\frac{32}{4} + 20 = -8 + 20 = \boxed{12}$

(b) What is the acceleration of the particle when $t = 2$?

$a(t) = v'(t) = -32\cdot(-2)t^{-3} + 10 = \frac{64}{t^3} + 10$

$a(2) = \frac{64}{8} + 10 = 8 + 10 = \boxed{18}$

6. (5 points) Find the point (x, y), at which the graph $y = 3x^2 + 3x - 10$ has a horizontal tangent.

$y' = 6x + 3 = 0$

$6x = -3$

$x = -1/2$

$y(-1/2) = \frac{3}{4} - \frac{3}{2} - 10$

$= \frac{3 - 6 - 40}{4} = \frac{-43}{4}$

$\boxed{\left(-\frac{1}{2}, -\frac{43}{4}\right)}$

instantaneous

7. (4 extra credit points) The derivative of a function represents the ~~average~~ rate of change of the function with respect to its variable. (true/false)

8. (4 extra credit points) Given a function $f(x)$, if the left-hand and right-hand limits as x approaches c exist and are equal then the limit as x approaches c exist. (true/false)