MTH130, Spring 2017

Exam #2

April 5, 2017

Name key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No graphing calculators are allowed!!

1. (10 points) Consider the function $f(x) = x^3 - 12x$.

(a) Find the interval(s) where the function is increasing and the interval(s) where it is decreasing.

$f'(x) = 3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

$x = \pm 2$

f'	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
	-3	0	3
	$+$	$-$	$+$
	$\nearrow$	$\searrow$	$\nearrow$

(b) Find the absolute minimum and the absolute maximum of f on $[0, 3]$

$f(0) = 0 \leftarrow$ maximum

$f(3) = -9$

$f(2) = -16 \leftarrow$ minimum

2. (10 points) Find the relative extrema, if any, of the function.

(a) $f(x) = \dfrac{1}{1+x^2} = (1+x^2)^{-1}$

$f'(x) = -(1+x^2)^{-2} \cdot 2x = \dfrac{-2x}{(1+x^2)^2}$

$-2x = 0$
$x = 0$

$1+x^2 = 0$
$x^2 = -1$
$\times$

$(-\infty, 0)$	$(0, \infty)$
-1	1
$+$	$-$
$\nearrow$	$\searrow$

$x=0$ is a local/relative maximum

(b) $f(t) = t^5 - 5t$

$f'(t) = 5t^4 - 5 = 0$
$5(t^4 - 1) = 0$
$t^4 = 1$
$t = \pm 1$

f'	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
	-2	0	2
	$+$	$-$	$+$
	$\nearrow$	$\searrow$	$\nearrow$

$t = -1$ relative max

$t = 1$ relative min

3. (15 points) Let $f(x) = 3x^4 + 4x^3$

(a) Find the interval(s) where the function is increasing and the interval(s) where it is decreasing.

$$f'(x) = 12x^3 + 12x^2 = 0$$
$$12x^2(x+1) = 0$$
$$x = 0$$
$$x = -1$$

$(-\infty, -1)$	$(-1, 0)$	$(0, \infty)$
$-$	$+$	$+$
$\searrow$	$\nearrow$	$\nearrow$

(b) Find the interval(s) where the function is concave up and the interval(s) where it is concave down.

$$f''(x) = 12 \cdot 3x^2 + 12 \cdot 2x = 0$$
$$12x(3x+2) = 0$$
$$x = 0$$
$$x = -\tfrac{2}{3}$$

$(-\infty, -\frac{2}{3})$	$(-\frac{2}{3}, 0)$	$(0, \infty)$
-1	$-\frac{1}{2}$	1
$+$	$-$	$+$
$\cup$	$\cap$	$\cup$

(c) Find the $x-$ and $y-$ intercepts.

$f(0) = 0 + 0 = 0 \rightarrow (0,0)$ is y-intercept

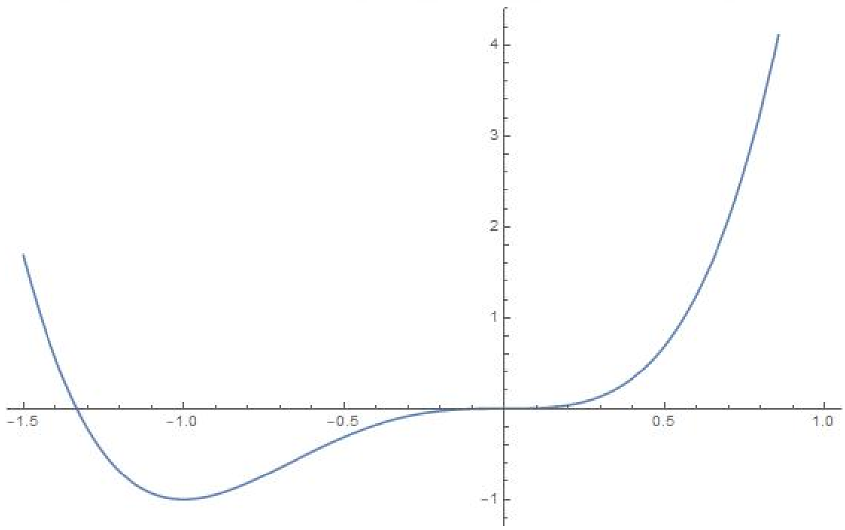
$$f(x) = 3x^4 + 4x^3 = 0$$
$$x^3(3x+4) = 0$$

$x = 0$

$x = -4/3$

$\left.\begin{matrix}(0,0)\\(-\frac{4}{3}, 0)\end{matrix}\right\}$ are x-intercepts

(d) Sketch the graph of $f(x)$. [Hint: Plot $f(-1), f(0), f(1)$ and the $x-$ and $y-$ intercepts first.]



4. (10 points) Find the interval(s) where the function is concave up and the interval(s) where it is concave down

$$f(x) = \frac{x}{x-3}$$

$$f'(x) = \frac{1\cdot(x-3) - 1\cdot x}{(x-3)^2} = \frac{-3}{(x-3)^2} = -3(x-3)^{-2}$$

$$f''(x) = 6(x-3)^{-3}\cdot 1 = \frac{6}{(x-3)^3}$$

$x - 3 = 0$

$\underline{x = 3}$

$(-\infty, 3)$	$(3, \infty)$
0	4
$-$	$+$
$\cap$	$\cup$

5. (10 points) Find the interest rate needed for an investment of \$5,000 to double in 15 years if interest is (a) compounded continuously, (b) compounded monthly.

(a)

$$10000 = 5000e^{r \cdot 15}$$

$$2 = e^{15r}$$

$$\ln 2 = 15r \cdot \ln e$$

$$r = \frac{\ln 2}{15} \approx 0.0462$$

$$\boxed{4.62\%}$$

(b)

$$10000 = 5000\left(1+\frac{r}{12}\right)^{12 \cdot 15}$$

$$2 = \left(1+\frac{r}{12}\right)^{180}$$

$$2^{\frac{1}{180}} = 1+\frac{r}{12}$$

$$1.00385 = 1+\frac{r}{12}$$

$$r/12 = 0.00385$$

$$r = 0.0463$$

$$\boxed{4.63\%}$$

6. (10 points) Find the accumulated amount after 5 years if \$10,000 is invested at 7%/year (a) compounded continuously, (b) compounded yearly.

(a)

$$A = 10000e^{5 \cdot 0.07}$$

$$= \boxed{14{,}190.675}$$

(b)

$$A = 10000 \cdot (1+0.07)^5$$

$$= \boxed{14{,}025.5}$$

7. (5 points) Use the laws of logarithms to expand and simplify the expression.

$$\ln \frac{x^2(x+3)}{e^3} = \ln x^2(x+3) - \ln e^3$$

$$= \ln x^2 + \ln(x+3) - 3 \cdot \ln e$$

$$= \boxed{2\ln x + \ln(x+3) - 3}$$

8. (10 points) Find the derivative of the function.

(a) $f(x) = e^{-3x}$

$$f'(x) = e^{-3x}(-3) = \boxed{-3e^{-3x}}$$

(b) $g(x) = \ln(3x^2 - 1)$

$$g'(x) = \frac{1}{3x^2-1} \cdot 6x = \boxed{\frac{6x}{3x^2-1}}$$

9*. (a) (**1 extra point**) Is it true that $\frac{a+b}{a} = \frac{\cancel{a}+b}{\cancel{a}} = \frac{1+b}{1}$ for any b and $a \neq 0$?

NO!

(b) (**5 extra points**) Find the absolute minimum of the function $f(x) = \frac{e^x}{x^2}$ on $[1, 4]$.

$f(x) = e^x \cdot x^{-2}$

$f'(x) = e^x x^{-2} - 2x^{-3} e^x$

$= x^{-3} e^x (x-2) = 0$

$x = 2$

$x = 0$

$f(1) = e \approx 2.71828$

$f(4) = e^4/16 \approx 3.4123$

$f(2) = e^2/4 \approx 1.8472$

Abs. min is at $x = 2$.

Honor Code: *On my honor, I have neither received nor given any aid during this examination.*

Signature: ______________________