MAC 2233, Fall 2017

# Exam #2

October 25, 2017

**Name** ____________________________

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

**No calculators are allowed!**

| | |
|---|---|
| Revenue function: | $R(x) = p * x$ |
| Profit function: | $P(x) = R(x) - C(x)$ |
| Elasticity of demand: | $E(p) = -\frac{p \cdot q'(p)}{q(p)}$ |
| Future value of an investment: | $B(t) = P(1 + \frac{r}{k})^{kt}$<br>$B(t) = Pe^{rt}$ |
| Effective interest: | $r_e = (1 + \frac{r}{k})^k - 1$<br>$r_e = e^r - 1$ |

**Honor Code:** On my honor, I have neither received nor given any aid during this examination.

Signature: ________________________

1. (15 points) Find the intervals where the function is increasing/decreasing, concave up/down and find the relative min/max and inflection points.

$$f(x) = \frac{x}{x^2+1}$$

$$\left[\text{Hint: } f''(x) = \frac{2x\left(x^2-3\right)}{\left(x^2+1\right)^3}\right]$$

$$f'(x) = \frac{(x^2+1)\cdot 1 - x\cdot 2x}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$1-x^2=0$

$1=x^2$

$x=\pm 1$

$x^2+1=0$

$x^2=-1$

none

| $f'$ | $(-\infty,-1)$ | $(-1,1)$ | $(1,\infty)$ |
|---|---|---|---|
| $1-x^2$ | $-$ | $+$ | $-$ |
| $(x^2+1)^2$ | $+$ | $+$ | $+$ |
| $f'$ | $-$ ↘ | $+$ ↗ | $-$ ↘ |

$\boxed{\begin{array}{l} x=-1 \text{ rel. min} \\ x=1 \text{ rel. max} \end{array}}$

$$= \frac{2x(x^2-3)}{(x^2+1)^3}$$

$2x=0$

$x=0$

$x^2-3=0$

$x=\pm\sqrt{3}$

| $f''$ | $(-\infty,-\sqrt{3})$ | $(-\sqrt{3},0)$ | $(0,\sqrt{3})$ | $(\sqrt{3},\infty)$ |
|---|---|---|---|---|
| $2x$ | $-$ | $-$ | $+$ | $+$ |
| $x^2-3$ | $+$ | $-$ | $-$ | $+$ |
| $(x^2+1)^3$ | $+$ | $+$ | $+$ | $+$ |
| $f''$ | $-$ ⌒ | $+$ ∪ | $-$ ⌒ | $+$ ∪ |

$\boxed{x=\pm\sqrt{3}, 0}$ are inflection pts

2. (10 points) Find the critical numbers of the given function and classify each as a relative minimum or maximum

$$f(x) = x^3(x-2)^2$$

$f'(x) = 3x^2(x-2)^2 + 2(x-2)x^3 = x^2(x-2)[3(x-2)+2x]$

$= x^2(x-2)(3x-6+2x) = x^2(x-2)(5x-6)$

$f'(x) = 0$ $\quad x^2 = 0$ $\quad x-2=0$ $\quad 5x-6=0$

$\quad x=0$ $\quad x=2$ $\quad x = 6/5$

$\boxed{x = \frac{6}{5} \text{ is a relative max}}$

$\boxed{x = 0, \frac{6}{5}, 2 \text{ are crit. num}}$

$\boxed{x = 2 \text{ is a rel. min}}$

| $f'$ | $(-\infty, 0)$ | $(0, \frac{6}{5})$ | $(\frac{6}{5}, 2)$ | $(2, \infty)$ |
|---|---|---|---|---|
| $x^2$ | + | + | + | + |
| $x-2$ | − | − | − | + |
| $5x-6$ | − | − | + | + |
| $f'$ | + | + | − | + |

3. (10 points) Find the intervals where the function is increasing/decreasing

(a) $f(x) = \dfrac{16}{x} + x^2 = 16x^{-1} + x^2$

$f'(x) = -16x^{-2} + 2x = 2x^{-2}(-8 + x^3)$

$x = 0$ $\quad x^3 = 8$

$x = 2$

| $f'$ | $(-\infty, 0)$ | $(0, 2)$ | $(2, \infty)$ |
|---|---|---|---|
| $\frac{2}{x^2}$ | + | + | + |
| $x^3-8$ | − | − | + |
| $f'$ | − ↘ | − ↘ | + ↗ |

4. (10 points) Find the elasticity of demand and determine whether the demand is elastic, inelastic, or unitary at the indicated price.

(a) $q(p) = 240 - 2p; p = 50$

$q'(p) = -2$ $\quad E(p) = \frac{2p}{240-2p} = \frac{p}{120-p}$

$E(50) = \frac{50}{70} = \boxed{\frac{5}{7}} < 1$ Demand is inelastic

5. (10 points) Differentiate the given function.

(a) $f(x) = e^{3x+1}$

$f'(x) = e^{3x+1} \cdot (3x+1)' = e^{3x+1} \cdot 3$

$= \boxed{3e^{3x+1}}$

(b) $f(x) = 3\log_5 x^2$

$f(x) = 3 \cdot 2 \log_5 x = 6 \log_5 x$

$f'(x) = \boxed{\frac{6}{x \cdot \ln 5}}$

6. (10 points) Use the marginal cost to estimate the cost of producing the 6th unit of a commodity if the cost function is

$$C(x) = \frac{1}{2}x^2 - 3x + 110$$

$$C'(x) = x - 3$$

$$C'(5) = 5 - 3 = \boxed{2}$$

7. (15 points) A citrus grower in Florida estimates that if 100 orange trees are planted, the average yield will be 60 oranges per tree. The average yield will decrease by 2 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plan to maximize the total yield? [Hint: Use $x$ to denote the number of orange trees and find the total yield as a function of $x$.]

| $x$ | yield per tree | total yield |
|---|---|---|
| 100 | $60$ | $100 \cdot 60$ |
| 101 | $60-2$ | $101 \cdot (60-2)$ |
| 102 | $60 - 2 \cdot (102-100)$ | $102 \cdot (60 - 2(102-100))$ |
| $x$ | $60-2(x-100) = 60-2x+200$ <br> $260-2x$ | $x(60-2(x-100)) = x(260-2x) = 260x - 2x^2$ |

$$T(x) = 260x - 2x^2$$

$$T'(x) = 260 - 4x = 0$$

$$x = \frac{260}{4} = \boxed{65}$$

$$T''(x) = -4$$

Hence $x = 65$ is a relative maximum.

8. (10 points) Differentiate [Hint: simplify first]

$$f(x) = \ln\left(x^7(x^2+3)^4\right)$$

$$= \ln x^7 + \ln(x^2+3)^4$$

$$= 7\ln x + 4\ln(x^2+3)$$

$$f'(x) = \frac{7}{x} + 4\cdot\frac{2x}{x^2+3} = \boxed{\frac{7}{x} + \frac{8x}{x^2+3}}$$

9. (5 extra credit points) Find the derivative of $f(x) = x^x$.

$$x^x = e^{x\ln x}$$

$$f'(x) = e^{x\cdot\ln x}\cdot(x\cdot\ln x)' = x^x\cdot\left(\ln x + x\cdot\frac{1}{x}\right)$$

$$= \boxed{x^x(1+\ln x)}$$

10. (5 extra credit points) Find the absolute minimum and maximum of the function $f(x) = \frac{e^x}{x}$ in the interval $[1, 4]$. [Hint: $e^4 \approx 54.6$]

$$f'(x) = \frac{e^x\cdot x - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$$

$e^x = 0$ none $\quad x-1=0$ $x=1$ $\quad x^2=0$ $x=0$

Since only $x=1$ is in $[1,4]$

we have to compare

$f(1)$ and $f(4)$.

$f(1) = \frac{e^1}{1} = e \approx 2.718$

$f(4) = \frac{e^4}{4} \approx \frac{54}{4} \approx 13$



$\boxed{\begin{array}{l}x=4 \text{ is abs. max} \\ x=1 \text{ is abs. min}\end{array}}$