MAC 2233, Fall 2017

Exam #3

November 20, 2017

Name Key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No calculators are allowed!

Future value of an income stream:	$FV = e^{rT} \int_0^T f(t)e^{-rt}\,\mathrm{d}t$
Useful lifetime:	$R'(t) = C'(t)$

Honor Code: On my honor, I have neither received nor given any aid during this examination.

Signature: _____

1. (10 points each) Find the indefinite integral.

(a) $\int 2 - 12x^3 \, dx$

$= 2x - 12 \cdot \frac{1}{4} x^4 + C$

$= \boxed{2x - 3x^4 + C}$

(b) $\int (x-1)(x+1) \, dx$ $= \int x^2 - 1 \, dx = \boxed{\frac{1}{3}x^3 - x + C}$

(c) $\int t^3(t^4-2)^4 \, dt$ $= \left| \begin{array}{l} u = t^4 - 2 \\ du = 4t^3 \, dt \\ \frac{1}{4} du = t^3 \, dt \end{array} \right| = \frac{1}{4} \int u^4 \, du$

$= \frac{1}{4} \cdot \frac{1}{5} u^5 + C$

$= \boxed{\frac{1}{20} (t^4 - 2)^5 + C}$

(d) $\int \frac{2}{x \ln(x)} \, dx$ $= \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array} \right| = 2 \int \frac{1}{u} \, du$

$= 2 \ln|u| + C$

$= \boxed{2 \ln|\ln x| + C}$

2. (15 points each) Evaluate the integral and simplify your answer.

(a) $\int_3^{11} \frac{1}{\sqrt{2x+3}}\,dx$

$= \left| \begin{array}{l} u = 2x+3 \\ du = 2\,du \\ \frac{1}{2}du = du \end{array} \right| = \frac{1}{2}\int_9^{25} \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_9^{25} u^{-1/2}\,du$

$x=3 \to u = 2\cdot 3+3 = 9$

$x=11 \to u = 2\cdot 11+3 = 25$

$= \frac{1}{2}\cdot\frac{2}{1}u^{1/2}\Big|_9^{25} = \sqrt{25} - \sqrt{9} = 5-3 = \boxed{2}$

(b) $\int_0^1 6x^2e^{x^3}\,dx$

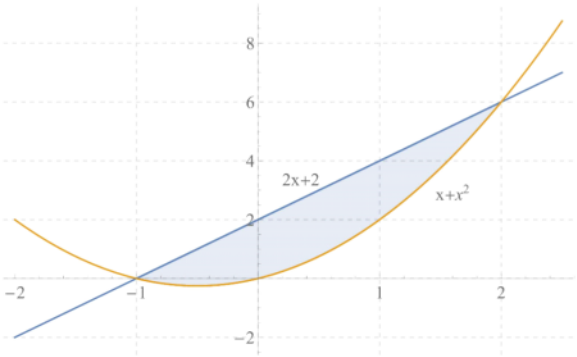
$= \left| \begin{array}{l} u = x^3 \\ du = 3x^2\,dx \\ \frac{1}{3}du = x^2dx \end{array} \right| = 6\cdot\frac{1}{3}\int_0^0 e^u\,du$

$= 2\,e^u\Big|_0^0$

$= 2\,e^{x^3}\Big|_0^1 = 2e^1 - 2e^0$

$= \boxed{2e-2}$

3. (5 points) Setup **but do not evaluate** the integral that represents the area of the shaded region.



$\int_{-1}^{2} 2x+2-(x+x^2)\,dx$

$= \boxed{\int_{-1}^{2} x+2-x^2\,dx}$

4. (5 points) Check that F is an antiderivative of f. [Hint: You have to differentiate a function.]

$$F(x) = x\ln(x) - x + 2; \quad f(x) = \ln(x)$$

$F'(x) = (x\ln x)' - 1 + 0 = 1\cdot\ln x + x\cdot\frac{1}{x} - 1$

$= \ln x + 1 - 1 = \ln x = f(x)$ ✓

5. (10 points) Find the average value of $f(x) = e^{3x}$ over the interval $[-1, 1]$.

Ave: $\frac{1}{1-(-1)}\cdot\int_{-1}^{1} e^{3x}\,dx = \frac{1}{2}\cdot\frac{1}{3}\,e^{3x}\Big|_{-1}^{1}$

$= \boxed{\frac{1}{6}\left(e^{3} - e^{-3}\right)}$

6. (10 points) At age 25, Alice starts making annual deposits of \$3500 into an IRA account that pays interest at an annual rate of 3% compounded continuously. Assuming the her payments are made as a continuous income flow, how much money will be in her account if she retires at the age of 65?

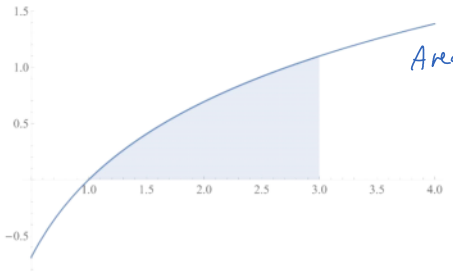
$FV = e^{0.03\cdot 40}\int_0^{40} 3500\,e^{-0.03t}\,dt$

$= e^{0.03\cdot 40}\cdot 3500\int_0^{40} e^{-0.03t}\,dt = e^{0.03\cdot 40}\,3500\,\frac{1}{-0.03}\,e^{-0.03t}\Big|_0^{40}$

$= \frac{-3500}{0.03}\,e^{0.03\cdot 40}\left(e^{-0.03\cdot 40} - e^{0}\right)$

$= \boxed{\frac{-3500}{0.03}\left(1 - e^{0.03\cdot 40}\right)} = \boxed{\frac{3500}{0.03}\left(e^{1.2} - 1\right)}$

7. (5 extra credit points) Find the area under the graph of $\ln(x)$ on the interval $(1, 3)$. The function is depicted below. [Hint: You already saw an antiderivative of $\ln(x)$.]



$$\text{Area} = \int_1^3 \ln x \, dx = x\ln x - x + 2 \Big|_1^3$$

$$= 3\ln 3 - 3 + 2 - (1\ln 1 - 1 + 2)$$

$$= 3\ln 3 - 1 - 0 + 1 - 2$$

$$= \boxed{-2 + 3\ln 3}$$

8. (2.5 extra credit points each) Determine if the following statement is true or false.

(a) (true / (false))

$$\int \frac{x^2}{x-1}\,dx = \frac{\frac{1}{3}x^3}{\frac{1}{2}x^2 - x} + C$$

↖ we cannot integrate the num. and denom. separately

(b) (true / (false))

$$\int_0^4 \frac{2x}{\sqrt{x^2-3}}\,dx = \left| \begin{array}{l} u = x^2 - 3 \\ du = 2x\,dx \end{array} \right| = \int_0^4 \frac{1}{\sqrt{u}}\,du = \int_0^4 u^{-1/2}\,du$$

$$= 2u^{1/2}\Big|_0^4 = 2\sqrt{4} - 2\sqrt{0} = 2 \cdot 2 = 4$$

↖ need to change the bounds or do back subs.