

Exam #4

December 6, 2017

Name _____

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No calculators are allowed!

$$D(x, y) = f_{xx}(x, y) \cdot f_{yy}(x, y) - \left(f_{xy}(x, y) \right)^2$$

Honor Code: On my honor, I have neither received nor given any aid during this examination.

Signature: _____

1. (15 pts each) Find the second partial derivatives (mixed derivative included)

(a) $f(x, y) = e^{xy}$

(b) $f(x, y) = \ln(x^2 + y)$

2. (15 pts) The demand function for peanut butter is

$$D_1(p_1, p_2) = 800 - 3p_1 - 4p_2$$

while that for a second commodity is

$$D_2(p_1, p_2) = 500 - 2p_2^3 - \frac{p_1}{2}$$

Is the second commodity more likely to be jelly or bread? Explain. [Hint: are the two commodities substitute or complementary?]

3. (15 pts) A grocer's daily profit from the sale of two brands of cat food is

$$P(x, y) = 5x^2 + 2xy - 460x - 7y^2 + 480y - 1100$$

dollars, where x is the price per can of the first brand and y is the price per can of the second. Currently the first brand sells for \$2 per can and the second for \$5 per can.

- (a) Use marginal analysis to estimate the change in daily profit that will result if the grocer raises the price of the **first** brand by \$1 per can but keeps the price of the second brand unchanged.

- (b) Use marginal analysis to estimate the change in daily profit that will result if the grocer raises the price of the **second** brand by \$1 per can but keeps the price of the first brand unchanged.

- (c) Increasing the price of which brand will yield higher daily profit?

4. (20 pts each) Find the critical points of the given function and classify each as a relative maximum, a relative minimum, or a saddle point.

(a) $f(x, y) = xy$

(b) $f(x, y) = x^3 - 12x + y^2 + 2y + 2$

5. (5 extra credit points) Find the critical points of the functions in problem 1 and classify each as a relative maximum, a relative minimum, or a saddle point. [Hint: $e^x = 0$ does not have a solution.]

(a) $f(x, y) = e^{xy}$

(b) $f(x, y) = \ln(x^2 + y)$

6. (2 extra credit points each) Determine if the following statement is true or false.

(a) (true / false)

A function $f(x, y)$ has a relative maximum at (a, b) if the sign of both first derivatives change from positive to negative about the point (a, b) .

(b) (true/false)

If a function $f(x, y)$ has all first and second partial derivatives $f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx}$ then $f_{xy} = f_{yx}$.