

No graphing calculators are allowed on this quiz. Please read each question carefully, follow directions and clearly mark your solutions. **Show your work for full credit.**

1. (8 points) Find the interval(s) where the function is increasing or decreasing and concave up or down. Does the function have any relative minimum or relative maximum?



(a) $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

$f''(x) = 0$

$6(x-1) = 0$
 $x = 1$

f''	$(-\infty, 1)$	$(1, \infty)$
	-	+
		

$f'(x) = 0$

$3x(x-2) = 0$

$x = 0, 2$

f'	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
$3x$	-	+	+
$x-2$	-	-	+
f'	+	-	+

$x = 0$ rel max
 $x = 2$ rel min

(b) $f(x) = \frac{x^2}{x+1}$



$f'(x) = \frac{(x+1)2x - x^2 \cdot 1}{(x+1)^2} = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$

$f''(x) = \frac{(x+1)^2 \cdot (2x+2) - (x^2+2x) \cdot 2(x+1)}{(x+1)^4} = \frac{(x+1)[(x+1)(2x+2) - 2(x^2+2x)]}{(x+1)^4}$

$= \frac{2x^2 + 2x + 2x + 2 - 2x^2 - 4x}{(x+1)^3} = \frac{2}{(x+1)^3}$

f'	$(-\infty, -2)$	$(-2, -1)$	$(-1, 0)$	$(0, \infty)$
	-	-	-	+
x	-	-	-	+
$(x+2)$	-	+	+	+
$(x+1)^2$	+	+	+	+
f'	+	-	-	+

$x = -2$ rel. max
 $x = 0$ rel. min

f''	$(-\infty, -1)$	$(-1, \infty)$
	-	+
$(x+1)^3$	-	+
f''	-	+
		

2. (2 points) $C(x)$ is the total cost of producing x units of a particular commodity. Assume $C(x)$ is in dollars.

$$C(x) = \frac{2}{3}x^3 - 2x + 3.$$

Find and interpret the marginal cost when $x = 10$.

$$C'(x) = 2x^2 - 2$$

$$C'(10) = 200 - 2 = 198$$

~~C~~ Producing the 11th unit will cost approximately \$198.