ws_3.1-3.3

Friday, October 6, 2017 10:11 AM

ws_3.1-3.3

MAC2233, Worksheet 10/6

1. Consider the function $f(x) = x^4 - 2x^2 + 1$.

   a) Find the intervals on which f is increasing or decreasing.

   b) Find the relative min/max of f.

   c) Find the intervals of concavity and the inflection points.

a) $f'(x) = 4x^3 - 4x = 0$

$\underbrace{4x}_{x=0}\underbrace{(x^2-1)}_{x=\pm 1} = 0$

f'	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
	-2	$-.5$	0.5	2
	$-$	$+$	$-$	$+$
	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

local min: $x=-1$, $x=1$

loc. max: $x=0$

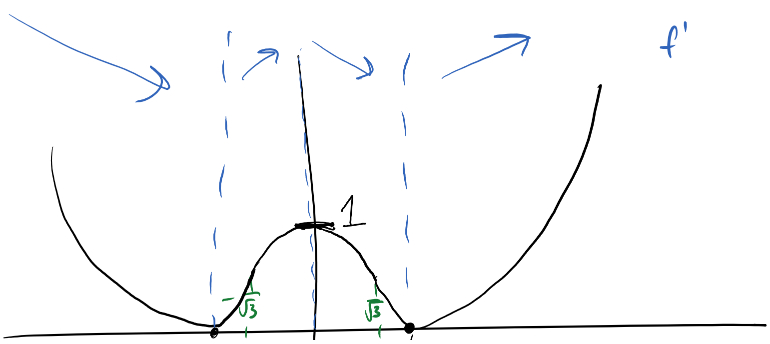
$f''(x) = 12x^2 - 4 = 0$

$4(3x^2-1) = 0$

$x = \pm\frac{1}{\sqrt{3}}$

f''	$(-\infty,-\frac{1}{\sqrt{3}})$	$(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})$	$(\frac{1}{\sqrt{3}},\infty)$
	-4	0	4
	$+$	$-$	$+$
	$\cup$	$\cap$	$\cup$

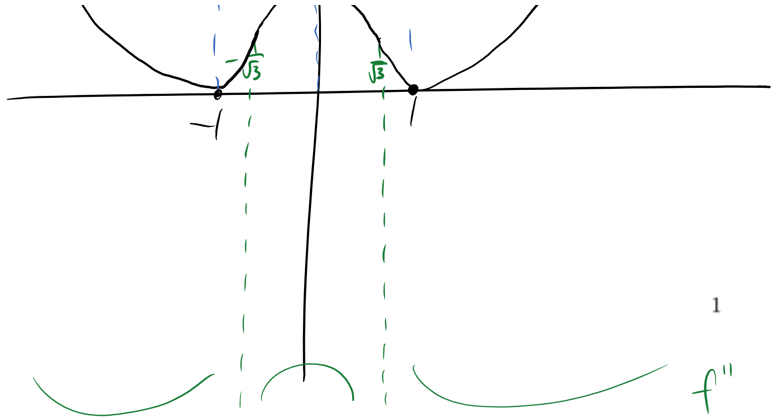
2. Use the previous problem to sketch the function $f(x) = x^4 - 2x^2 + 1$.



$f(-1) = 1-2+1 = 0$

$f(1) = 1-2+1 = 0$

$f(0) = 1$



$f(0) = 1$

1

3. Consider the function $f(x) = \frac{x}{\sqrt{x^2+1}}$. Find the following:

   Domain, intercepts, symmetry, asymptotes(horizontal and vertical), intervals of increase or decrease, local min/max, concavity and points of inflection. Use the data to sketch the curve.

$x^2+1=0$ DNE , $f(0)=0$, $f(x)=0 \Leftrightarrow x=0$

Domain: $\mathbb{R}$

intercept: $(0,0)$

$f(-x) = \frac{-x}{\sqrt{x^2+1}} = -f(x) \Rightarrow$ odd

$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} \cdot \frac{\frac{1}{x}}{\frac{1}{\sqrt{x^2}}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = \frac{1}{1} = \boxed{1}$ $\Big|\Big|$ $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}} \cdot \frac{\frac{1}{x}}{\frac{-1}{\sqrt{x^2}}} = \lim_{x\to-\infty} \frac{-1}{\sqrt{1+\frac{1}{x^2}}} = \boxed{-1}$

hor. asym: $y = \pm 1$

$f'(x) = \frac{(x^2+1)^{1/2} - x\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x}{x^2+1} = \frac{(x^2+1)^{-1/2}\left[(x^2+1)-x^2\right]}{(x^2+1)} = \frac{(x^2+1)^{-1/2}\cdot 1}{(x^2+1)^{\frac{2}{2}}} = (x^2+1)^{-3/2}$

$f''(x) = -\frac{3}{2}(x^2+1)^{-5/2}\cdot(2x) = \frac{-3x}{(x^2+1)^{5/2}}$

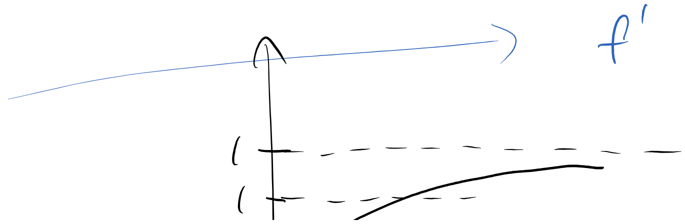
$f'(x)=0$ ✗

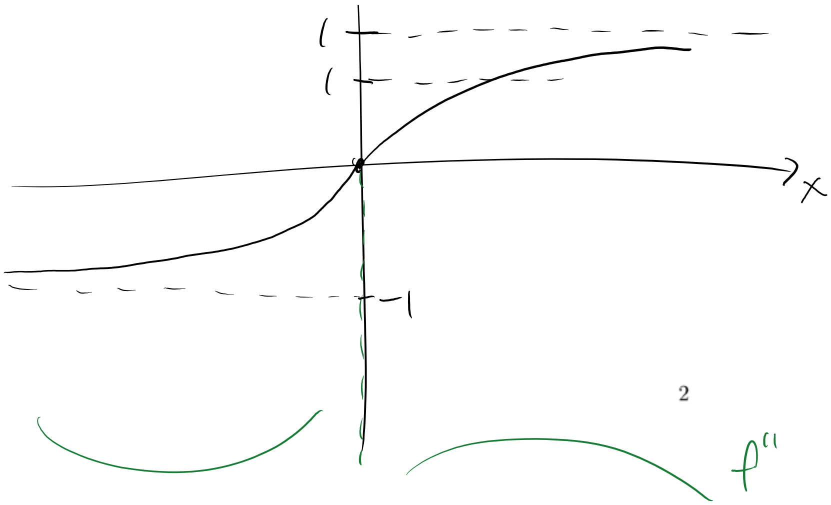
f'	$(-\infty,\infty)$
0	+ ↗

f''	$(-\infty,0)$	$(0,\infty)$
	-1 + ∪	1 − ∩

$-3x=0$

$\boxed{x=0}$





1
1
x
-1
2
f''

4. Find the critical numbers of the function

a) $g(x) = x^{\frac{1}{3}} - x^{\frac{-2}{3}}$

$$g'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3} = \underbrace{\frac{1}{3}x^{-5/3}}_{x=0}\underbrace{\left[x^{3/3} + 2\right]}_{\substack{x+2=0 \\ x=-2}} = 0$$

Since $x=0$ is <u>not</u> in the domain, the only crit. number is $\boxed{x=-2}$

b) $f(x) = 1 + (x-3)^2$ on $(-2, 3]$

$f'(x) = 2(x-3)$

$x - 3 = 0$

$\boxed{x=3}$