Thm: (The second derivative test)

Suppose $f''(x)$ exists on an open interval containing $x=c$, and $f'(c)=0$ then

- $x=c$ is a relative max if $f''(c)<0$
- $x=c$ is a relative min. if $f''(c)>0$

Ex: Find relative extrema of:

$$f(x) = 2x^3 + 3x^2 - 12x - 7$$

$$f'(x) = 6x^2 + 6x - 12 = 0$$

$$6(x^2 + x - 2) = 0$$

$$6(x-1)(x+2) = 0$$

$x=1,\ x=-2$ } crit. values

$$f''(x) = 12x + 6$$

$f(1) = 12+6 = 18 > 0 \rightarrow x=1$ is a relative min

$f(-2) = -24+6 = -18 < 0 \rightarrow x=-2$ is a rel. max.

Section 3.3

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Def: The line $x=c$ is a vertical asymptote if both of these conditions hold:

$$\lim_{x\to c^-} f(x) = \infty \quad (\text{or } -\infty)$$

$$\lim_{x\to c^+} f(x) = \infty \quad (\text{or } -\infty)$$

Def: The line $y=c$ is a hor. asymp. if either $\lim_{x\to\infty} f(x)=c$ or $\lim_{x\to-\infty} f(x)=c$.

Ex: Sketch the function $f(x)=\dfrac{x}{(x+1)^2}$

Dom: $(-\infty,-1)\cup(-1,\infty)$

Ver. as: $(x+1)^2=0$

$$x=-1$$

Hor. as:

$$\lim_{x\to\infty} \frac{x}{(x+1)^2} = \lim_{x\to\infty} \frac{x}{x^2+2x+1} = 0$$

$$\lim \frac{x}{x^2+2x+1} = 0$$

$$\lim_{x\to-\infty} \frac{x}{x^2+2x+1} = 0$$

Inc/ dec:

$(x+1)'$

$$f'(x) = \frac{(x+1)^2\cdot 1 - x\cdot 2(x+1)\cdot 1}{\left[(x+1)^2\right]^2} = \frac{(x+1)^2 - 2x(x+1)}{(x+1)^4}$$

$$= \frac{\cancel{(x+1)}\left[x+1-2x\right]}{(x+1)^{\cancel{4}\,3}} = \frac{1-x}{(x+1)^3}$$

$1-x=0$ $\qquad$ $(x+1)^3=0$

$x=1$ $\qquad$ $x=-1$

-1 $\qquad$ 1

f'	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
	-2	0	2
$1-x$	$+$	$+$	$-$
$(x+1)^3$	$-$	$+$	$+$
	$-$	$+$	$-$
	$\searrow$	$\nearrow$	$\searrow$

$$f'(x) = \frac{1-x}{(x+1)^3}$$

$$f''(x) = \frac{(x+1)^3\cdot(-1) - (1-x)\,3(x+1)^2\cdot 1}{\left[(x+1)^3\right]^2} = \frac{-(x+1)^3 - 3(1-x)(x+1)^2}{(x+1)^6}$$

$$= \frac{\cancel{(x+1)^2}\left[-(x+1) - 3(1-x)\right]}{(x+1)^{\cancel{6}\,4}} = \frac{-x-1-3+3x}{(x+1)^4}$$

$$= \frac{2x-4}{}$$

$2x-4=0$ $\qquad$ $x+1=0$

$$= \frac{2x-4}{(x+1)^4}$$

$2x-4=0$

$x = \frac{4}{2} = 2$

$x+1=0$

$x=-1$

f''	$(-\infty,-1)$	$(-1,2)$	$(2,\infty)$
$2x-4$	$-$	$-$	$+$
$(x+1)^4$	$+$	$+$	$+$
	$-$	$-$	$+$
	$\cap$	$\cap$	$\cup$

$x=2 \leftrightarrow$ inflection pt.

$$f(0) = \frac{0}{(0+1)^2} = 0$$

$$f(1) = \frac{1}{(1+1)^2} = \frac{1}{4}$$

$$f(2) = \frac{2}{(2+1)^2} = \frac{2}{9}$$