

Thm: (The second derivative test)

Suppose $f''(x)$ exists on an open interval containing $x=c$, and $f'(c)=0$ then

- $x=c$ is a relative max if $f''(c) < 0$
- $x=c$ is a relative min. if $f''(c) > 0$

Ex: Find relative extrema of:

$$f(x) = 2x^3 + 3x^2 - 12x - 7$$

$$f'(x) = 6x^2 + 6x - 12 = 0$$

$$6(x^2 + x - 2) = 0$$

$$6(x-1)(x+2) = 0$$

$$x=1, x=-2 \quad \left. \vphantom{x=1, x=-2} \right\} \text{crit. values}$$

$$f''(x) = 12x + 6$$

$$f'(1) = 12 + 6 = 18 > 0 \rightarrow x=1 \text{ is a relative min}$$

$$f'(-2) = -24 + 6 = -18 < 0 \rightarrow x=-2 \text{ is a rel. max.}$$

Section 3.3

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Def: The line $x=c$ is a vertical asymptote if both of these conditions hold:

$$\lim_{x \rightarrow c^-} f(x) = \infty \text{ (or } -\infty)$$

$$\lim_{x \rightarrow c^+} f(x) = \infty \text{ (or } -\infty)$$

Def: The line $y=c$ is a hor. asymp. if either $\lim_{x \rightarrow \infty} f(x) = c$ or $\lim_{x \rightarrow -\infty} f(x) = c$.

Ex: Sketch the function $f(x) = \frac{x}{(x+1)^2}$

$$\text{Dom: } (-\infty, -1) \cup (-1, \infty)$$

Ver. as: $(x+1)^2 = 0$
 $x = -1$

Hor. as: $\lim_{x \rightarrow \infty} \frac{x}{(x+1)^2} = \lim_{x \rightarrow \infty} \frac{x}{x^2 + 2x + 1} = 0$

$$\lim_{x \rightarrow -\infty} \frac{x}{x^2 + 2x + 1} = 0$$

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Incl/dec:

$$f'(x) = \frac{(x+1)^2 \cdot 1 - x \cdot 2(x+1) \cdot 1}{[(x+1)^2]^2} = \frac{(x+1)^2 - 2x(x+1)}{(x+1)^4}$$

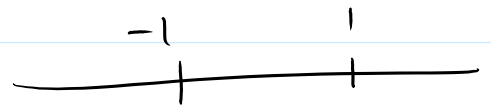
$$= \frac{\cancel{(x+1)} [x+1 - 2x]}{(x+1)^{\cancel{3} 3}} = \frac{1-x}{(x+1)^3}$$

$$1-x=0$$

$$\underline{x=1}$$

$$(x+1)^3=0$$

$$\underline{x=-1}$$



f'	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
$1-x$	+	+	-
$(x+1)^3$	-	+	+
	-	+	-

$$f'(x) = \frac{1-x}{(x+1)^3}$$

$$f''(x) = \frac{(x+1)^3 \cdot (-1) - (1-x) \cdot 3(x+1)^2 \cdot 1}{[(x+1)^3]^2} = \frac{-(x+1)^3 - 3(1-x)(x+1)^2}{(x+1)^6}$$

$$= \frac{\cancel{(x+1)}^2 [- (x+1) - 3(1-x)]}{(x+1)^{\cancel{6} 4}} = \frac{-x-1-3+3x}{(x+1)^4}$$

$$= \frac{2x-4}{(x+1)^4}$$

$$2x-4=0$$

$$x+1=0$$

$$= \frac{2x-4}{(x+1)^4}$$

$$2x-4=0$$

$$x = \frac{4}{2} = 2$$

$$x+1=0$$

$$x = -1$$



f''	$(-\infty, -1)$	$(-1, 2)$	$(2, \infty)$
$2x-4$	-	-	+
$(x+1)^4$	+	+	+
	∩	∩	∪

$x=2 \Leftrightarrow$ inflection pt.

$$f(0) = \frac{0}{(0+1)^2} = 0$$

$$f(1) = \frac{1}{(1+1)^2} = \frac{1}{4}$$

$$f(2) = \frac{2}{(2+1)^2} = \frac{2}{9}$$

