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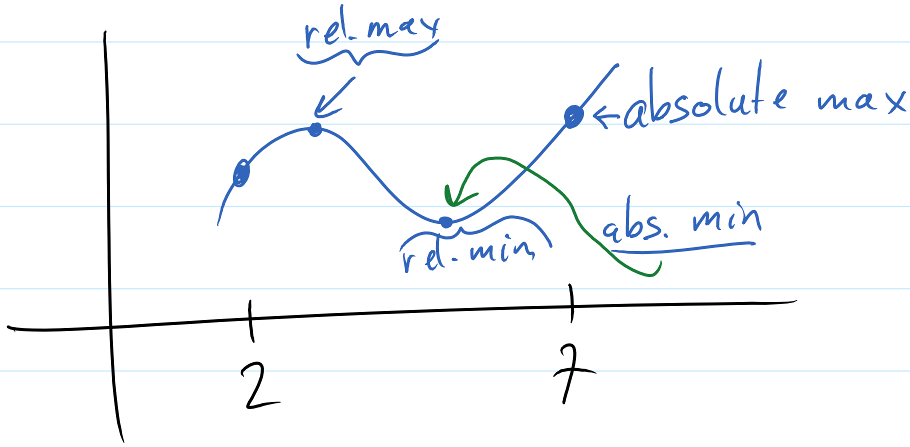
Tuesday, October 10, 2017 1:50 PM

HW for 3.3 - 4.2
Quiz for 3.3-3.5 } posted

Section 3.4 (optimization, elasticity of demand)



Note: An absolute min/max can be attained at relative extrema or at the endpts of an interval.

Thm: (The Extreme Value Property)
A function $f(x)$ that is continuous on $[a,b]$ attains absolute extrema on the interval $[a,b]$.

Ex: $f(x) = 2x^3 + 3x^2 - 12x - 7$, find the abs min/max on $-3 \le x \le 0$.

$$f'(x) = 6x^2 + 6x - 12$$

$$f'(x) = 0$$

$$6(x^2 + x - 2) = 0$$

$$6(x-1)(x+2) = 0$$

$$\underbrace{x=1}_{\text{not in } [-3,0]},\ \underbrace{x=-2} \leftarrow \text{crit. numbers}$$

evaluate $f(x)$ at crit. numbers in the interval and the endpts.

x	-2	-3	0
$f(x)$	13	2	-7

abs. max (13) ← abs. min (-7)

$$f(x) = 2x^3 + 3x^2 - 12x - 7$$

$$f(0) = -7$$

$$f(-3) = 2\cdot(-27) + 3\cdot 9 - 12(-3) - 7$$
$$= -54 + 27 + 36 - 7$$
$$= 2$$

$$f(-2) = 2\cdot(-8) + 3\cdot 4 + 24 - 7$$
$$= -16 + 12 + 24 - 7$$
$$= 20 - 7 = 13$$

abs. min is -7, abs. min. is at $x=0$

Ex: $f(x) = x^2 + \frac{16}{x}$ find abs. min/max on $x>0$.

$$= x^2 + 16x^{-1}$$

Ex: $f(x) = x^2 + \frac{16}{x}$ find abs. min/max on $x>0$.

$= x^2 + 16x^{-1}$

$$f'(x) = 2x - 16x^{-2} = 2x - \frac{16}{x^2} = \frac{2xx^2}{x^2} - \frac{16}{x^2} = \frac{2x^3-16}{x^2}$$

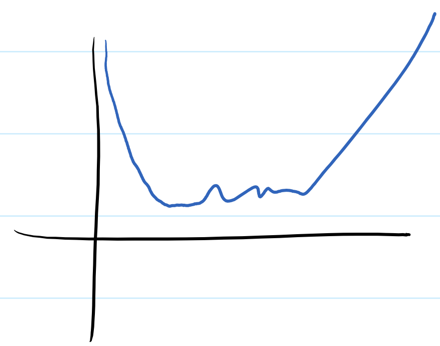
Find crit. num:

$$2x^3 - 16 = 0 \qquad x^2 = 0$$
$$2(x^3-8) = 0 \qquad \underline{x=0,}$$
$$x^3 = 8$$
$$\boxed{\underline{x=2,}}$$

interval: $(0,\infty)$

$$\lim_{x\to 0^+} x^2 + \frac{16}{x} = 0^2 + \frac{16}{0^+} = +\infty$$

$$\lim_{x\to\infty} x^2 + \frac{16}{x} = \infty^2 + 0 = \infty$$



To show that $x=2$ is an abs. min., make the table for f'.

f'	$(0,2)$	$(2,\infty)$
$2(x^3-8)$	$-$	$+$
x^2	$+$	$+$
f'		

$x=2$ is a rel. min and also an abs. min b/c $\lim f(x) = \infty$

x		
f'	$-$	$-$
	$\searrow$	$\nearrow$

min b/c $\lim_{x \to 0^+} f(x) = \infty$

and $\lim_{x \to \infty} f(x) = \infty$

There is no abs. max.