10/17

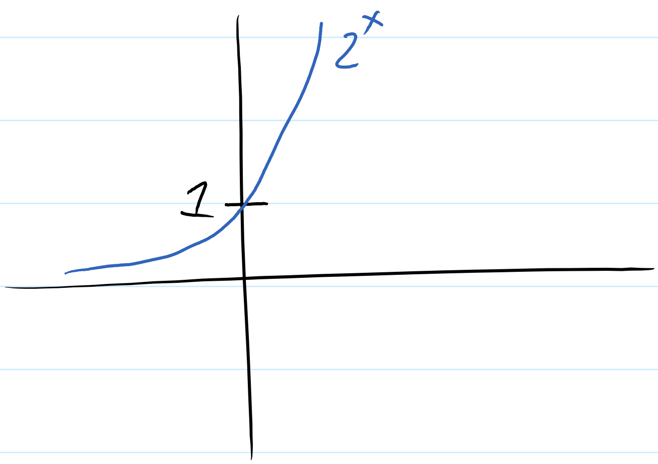
Tuesday, October 17, 2017 2:00 PM

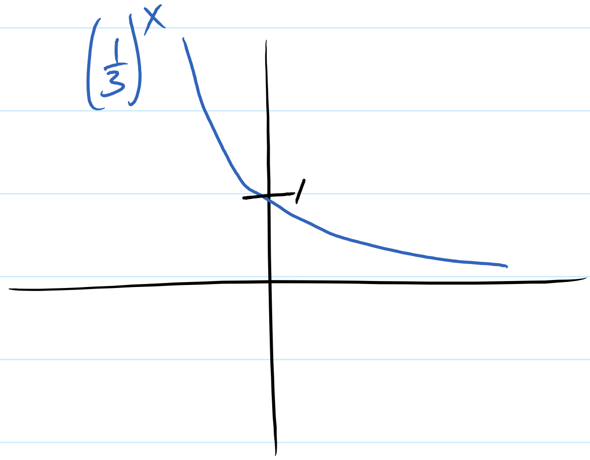
Test 2: Tuesday 10/24

online review on Monday evening

Section 4.1

Exponential func: 2^x, 4^x, 3^{2x}, $\left(\frac{1}{2}\right)^x$





$e \approx 2.718\ldots$

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Future Value of an Investment

Suppose a principal P is invested at an annual interest rate r for t years

at an annual interest rate r for t years to accumulate a future value $B(t)$.
If interest is compounded k times per year, then

$$B(t) = P\left(1+\frac{r}{k}\right)^{kt}$$

and

if interest is compounded continuously:

$$B(t) = P\, e^{rt}$$

Ex: Suppose \$1000 is invested with the interest rate 6% for 10 years. Find the future value if the interest is compounded:

a) Quarterly ($k=4$)

$$B(10) = 1000\left(1+\frac{0.06}{4}\right)^{4\cdot 10} \approx 1814.02$$

b) Monthly ($k=12$)

$$B(10) = 1000\left(1+\frac{0.06}{12}\right)^{12\cdot 10} \approx 1819.40$$

c) continuously

$$B(10) = 1000\, e^{0.06\cdot 10} = 1000\, e^{0.6} \approx 1822.12$$

Effective Interest Rate Formulas

If interest is compounded at the nominal rate r, the effective interest rate is the simple annual interest rate r_e that yields the same interest after 1 year.

If the compounding is k times per year, then:

$$r_e = \left(1+\frac{r}{k}\right)^k - 1,$$

if the interest is compounded continuously:

$$r_e = e^r - 1$$

Ex: Which is better, an investment that earns ① 10% compounded quarterly, one that earns ② 9.95% comp. monthly, or one that earns ③ 9.9% compounded continuously?

① $r_e = \left(1+\frac{0.1}{4}\right)^4 - 1 \approx 0.10381$

② $r_e = \left(1+\frac{0.0995}{12}\right)^{12} - 1 \approx 0.10416$ (the best)

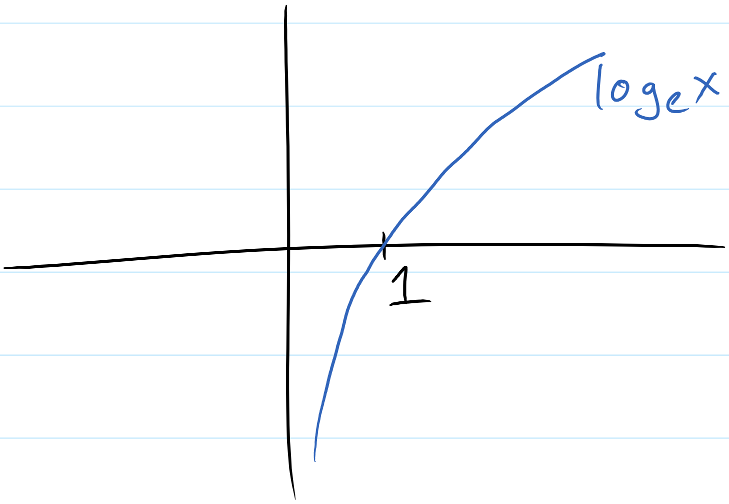
③ 0.099 ...

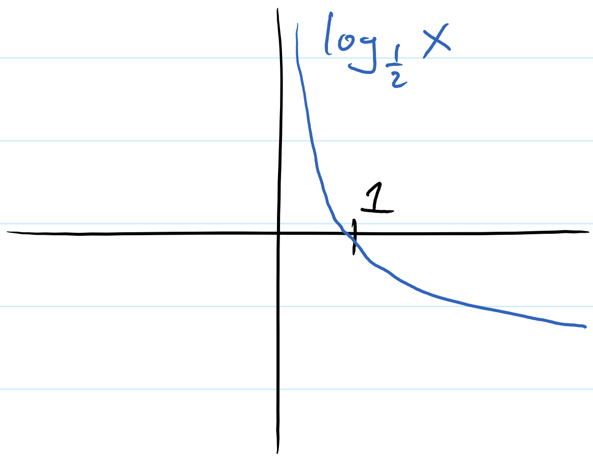
③ $r_e = e^{0.099} - 1 = 0.10406$

Section 4.2

log. functions : $\log_4 x$, $\log_{10} x$, $\log_e x$

$\log_{\frac{1}{2}} x$, $\log_{\frac{1}{3}} x$





• $\log_e x = \ln x$

• $\log_b 1 = 0$

• $\log_b x = c \longleftrightarrow b^c = x$

$\log_{10} 1000 = 3$

- $\log_b x^a = a \cdot \log_b x$
- $\log_b (x \cdot y) = \log_b x + \log_b y$
- $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

- $\log_e e^x = x \cdot \log_e e = x$
- $e^{\ln x} = x \quad , \quad (x>0)$
- $\log_b x = \frac{\log_c x}{\log_c b} = \frac{\ln x}{\ln b}$

Ex: How long will it take \$5000 to grow to \$7000 in an investment with the interest rate 6% if the compounding is

a) Quarterly $\qquad B(t) = P\left(1 + \frac{r}{k}\right)^{kt}$

need to solve:

$$7000 = 5000\left(1 + \frac{0.06}{4}\right)^{4t}$$

$$7/5 = \left(1 + \frac{0.06}{4}\right)^{4t} \qquad // \log$$

$$\frac{7}{5} = \left(1 + \frac{0.06}{4}\right)^{\cdots} \quad // \log$$

$$\log \frac{7}{5} = \log (1.015)^{4t}$$

$$\log \frac{7}{5} = 4t \log 1.015$$

$$t = \boxed{\frac{\log 7/5}{4 \log 1.015}} \approx 5.65$$

b) Continuous

$$7000 = 5000\, e^{0.06 \cdot t}$$

$$\frac{7}{5} = e^{0.06t} \quad // \ln$$

$$\ln 7/5 = 0.06t \cdot \ln e$$

$$t = \boxed{\frac{\ln 7/5}{0.06}} = \boxed{\frac{\log 7/5}{0.06 \cdot \log e}} \approx 5.61$$

Ex: A certain quantity grows exponentially,

$$Q(t) = Q_0\, e^{kt}$$

Q_0: initial value

(exp. decay: $Q(t) = Q_0 e^{-kt}$)

find how long does it take for this quantity to tripple.

solve: $3Q_0 = Q_0 e^{kt}$ for t.

$$\frac{3Q_0}{Q_0} = \frac{Q_0}{Q_0} e^{kt}$$

$$3 = e^{kt} \qquad // \ln$$

$$\ln 3 = kt \underbrace{\ln e}_{=1}$$

$$\boxed{t = \frac{\ln 3}{k}}$$

Section 4.3

Def:
$$\begin{cases} \frac{d}{dx}(e^x) = e^x \\ (e^x)' = e^x \end{cases}$$

$$\begin{cases} \frac{d}{dx}(\ln x) = \frac{1}{x} \\ (\ln x)' = \frac{1}{x} \end{cases}$$

Ex:

find $f'(x)$ given:

- $f(x) = x^2 e^x$

$$f'(x) = e^x \cdot (x^2)' + x^2 \cdot (e^x)'$$
$$= \boxed{e^x \cdot 2x + x^2 \cdot e^x}$$
$$= \boxed{x e^x (2+x)}$$

- $f(x) = \dfrac{x^3}{e^x + 2}$

$$f'(x) = \frac{(e^x+2) \cdot 3x^2 - x^3 \cdot (e^x)}{(e^x+2)^2}$$
$$= \frac{x^2\left[(e^x+2)3 - x e^e\right]}{(e^x+2)^2}$$
$$= \boxed{\frac{x^2(3e^x + 6 - x e^x)}{(e^x+2)^2}}$$

Chain rule:

$$\frac{d}{dx}\left(e^{u(x)}\right) = e^{u(x)} \cdot \frac{d}{dx} u(x)$$

$$\frac{d}{dx}\left(\ln u(x)\right) = \frac{1}{u(x)} \cdot \frac{d}{dx} u(x)$$

Ex: Differenciate:

- $$f(x) = \frac{e^{-3x}}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)\cdot\left(e^{-3x}\right)' - e^{-3x}\cdot\left(x^2+1\right)'}{(x^2+1)^2} = \frac{(x^2+1)\cdot e^{-3x}\cdot \overset{(-3x)'}{(-3)} - e^{-3x}\cdot 2x}{(x^2+1)^2}$$

$$= \boxed{\frac{e^{-3x}\left[-3x^2-3-2x\right]}{(x^2+1)^2}}$$

Ex: Find the abs. min/max of

$$f(x) = xe^{2x} \quad \text{on} \quad [-1,1].$$

$$f'(x) = e^{2x}\cdot 1 + xe^{2x}\cdot \overset{(2x)'}{2} = 0$$

$$e^{2x}(1+2x) = 0$$

$$e^{2x} = 0 \qquad\qquad 1+2x = 0$$

no sol. $\underline{x = -\frac{1}{2}}$

x	$f(x) = x e^{2x}$
-1	$-1\,e^{-2} = -e^{-2} = \frac{-1}{e^2} \approx \frac{-1}{7.3} \approx -0.13$
1	$1 \cdot e^2 = e^2$
$-\frac{1}{2}$	$-\frac{1}{2} e^{-2\cdot\frac{1}{2}} = -\frac{1}{2} e^{-1} = \frac{-1}{2e} \approx \frac{-1}{5.4} \approx -0.18$

min: at $x = -\frac{1}{2}$

max: at $x = 1$

Ex: Differenciate:

- $f(x) = x \cdot \ln x$

$$f'(x) = \ln x \cdot 1 + x \cdot \frac{1}{x} = \boxed{\ln x + 1}$$

- $$f(x) = \frac{\ln \sqrt[3]{x^2}}{x^4} = \frac{\ln x^{\frac{2}{3}}}{x^4} = \frac{\frac{2}{3}\ln x}{x^4}$$

$$= \frac{2}{3} \cdot \frac{\ln x}{x^4}$$

$$\boxed{\ln(x^2+1) \neq 2\ln(x+1)}$$

$$f'(x) = \frac{2}{3} \cdot \left(\frac{\ln x}{x^4}\right)'$$

$$= \frac{2}{3} \cdot \frac{x^4 \cdot \frac{1}{x} - \ln x \cdot 4x^3}{x^8}$$

$= \frac{2}{3} \cdot \frac{\quad}{x^8}$

$= \frac{2}{3} \cdot \frac{x^3 - 4x^3 \ln x}{x^8} = \frac{2}{3} \frac{\cancel{x^3}(1 - 4\ln x)}{x^{\cancel{8}\,5}}$

$= \boxed{\frac{2}{3} \frac{(1 - 4\ln x)}{x^5}}$

crit. pts:

$x^5 = 0$

$x = 0$

$1 - 4\ln x = 0$

$\ln x = \frac{1}{4}$

$e^{\ln x} = e^{1/4}$

$x = e^{1/4}$

0 not in domain, so the crit. pt is $\boxed{e^{1/4}.}$