10/19

Thursday, October 19, 2017 2:00 PM

online review on Sunday 6:45 PM

Section 4.3

Differentiate:

- $f(t) = (t + \ln t)^{3/2}$

$$f'(t) = \frac{3}{2}(t+\ln t)^{\frac{1}{2}} \cdot (t + \ln t)'$$

$$= \boxed{\frac{3}{2}(t+\ln t)^{1/2} \cdot \left(1 + \frac{1}{t}\right)}$$

$\ln x^a = a \cdot \ln x$

$\ln(x \cdot y) = \ln x + \ln y$

- $g(x) = \dfrac{\ln \sqrt[3]{x^2}}{x^4} = \dfrac{\ln x^{3/2}}{x^4}$

$$= \frac{3}{2} \cdot \frac{\ln x}{x^4}$$

$$g'(x) = \frac{3}{2} \cdot \left(\frac{\ln x}{x^4}\right)' = \frac{3}{2} \cdot \frac{x^4 \cdot \frac{1}{x} - \ln x \cdot (4x^3)}{x^8}$$

$$= \frac{3}{2} \cdot \frac{x^3 - 4x^3 \ln x}{x^8} = \frac{3}{2} \, \frac{\cancel{x^3}(1 - 4\ln x)}{\cancel{x^8}\ 5}$$

$$= \boxed{\frac{3}{2} \cdot \frac{1 - 4\ln x}{x^5}} = \boxed{\frac{3 - 12\ln x}{2x^5}}$$

Def:

$$\begin{cases} \dfrac{d}{dx}\left(\ln u(x)\right) = \dfrac{1}{u(x)} \cdot \dfrac{d}{dx} u(x) \\[2ex] \left(\ln u(x)\right)' = \dfrac{1}{u(x)} \cdot u'(x) \end{cases}$$

Diff:

- $f(x) = \ln(2x^3+1)$

$$f'(x) = \frac{1}{2x^3+1} \cdot (2x^3+1)' = \frac{1}{2x^3+1} \cdot (6x^2+0)$$

$$= \boxed{\frac{6x^2}{2x^3+1}}$$

$$\boxed{\ln \frac{x}{y} = \ln x - \ln y}$$

- $g(x) = \ln\left(\dfrac{x^2-2}{x^2+3x}\right)$

$$g(x) = \ln(x^2-2) - \ln(x^2+3x)$$

$$g'(x) = \frac{1}{\frac{x^2-2}{x^2+3x}} \cdot \left(\frac{x^2-2}{x^2+3x}\right)'$$

$$g'(x) = \frac{(x^2-2)'}{x^2-2} - \frac{(x^2+3x)'}{x^2+3x}$$

$$= \boxed{\frac{x^2+3x}{x^2-2} \cdot \frac{(x^2+3x)\cdot 2x - (x^2-2)\cdot(2x+3)}{(x^2+3x)^2}} = \boxed{\frac{2x}{x^2-2} - \frac{2x+3}{x^2+3x}}$$

How to diff. 2^x or $\log_2 x$?

$$2 = e^{\ln 2}$$

$$\boxed{\begin{aligned} e^{\ln x} &= x \\ \ln e^x &= x \end{aligned}}$$

$$(2^x)' = \left((e^{\ln 2})^x\right)' = \left(e^{x\ln 2}\right)'$$

$$= e^{x\ln 2} \cdot (x \cdot \ln 2)'$$

$\ln 2$ is a constant

$$= e^{x\ln 2} \cdot \ln 2 \cdot (x)'$$

$$= e^{x\ln 2} \cdot \ln 2$$

$$= 2^x \cdot \ln 2$$

Def:

$$\frac{d}{dx}(b^x) = b^x \cdot \ln b \quad , \quad \text{where } b>0, \ b \neq 1$$

$$\frac{d}{dx}(\log_b x) = \frac{d}{dx}\left(\frac{\ln x}{\ln b}\right)$$

$$= \frac{1}{\ln b} \cdot \frac{1}{x}$$

$$\boxed{\log_y x = \frac{\log_b x}{\log_b y}}$$

look up $(x^x)' = \frac{d}{dx}(x^x) = \ldots$

Ex: differenciate:

$$f(x) = 5^{2x-3}$$

$\cdot\ f(x) = 5^{2x-3}$

$$f'(x) = 5^{2x-3} \cdot \ln 5 \cdot (2x-3)'$$
$$= \boxed{2 \cdot \ln 5 \cdot 5^{2x-3}}$$

$\cdot\ g(x) = (x^2 + \log_7 x)^4$

$$g'(x) = 4(x^2 + \log_7 x)^3 \cdot \overbrace{\left(2x + \frac{1}{x \cdot \ln 7}\right)}^{(x^2 + \log_7 x)'}$$

$\cdot\ h(t) = \log_5(t^2+3)$

$$h'(t) = \frac{1}{t^2+3} \cdot (t^2+3)' \cdot \frac{1}{\ln 5}$$
$$= \boxed{\frac{2t}{(t^2+3)\ln 5}}$$

Ex: (logarithmic differentiation)

$$y = \frac{x^2(x-3)}{(x^3+2x)(x-1)}$$ // apply ln

$$\ln y = \ln\left(\frac{x^2(x-3)}{(x^3+2x)(x-1)}\right)$$

$$\ln y = \ln\left(\frac{x^2(x-3)}{(x^3+2x)(x-1)}\right)$$

$$= \ln\left(x^2(x-3)\right) - \ln\left((x^3+2x)(x-1)\right)$$

$$= \ln x^2 + \ln(x-3) - \left[\ln(x^3+2x) + \ln(x-1)\right]$$

$$\ln y(x) = 2\ln x + \ln(x-3) - \ln(x^3+2x) - \ln(x-1)$$

differenciate both sides:

$$\frac{1}{y(x)}\cdot y'(x) = \frac{2}{x} + \frac{1}{x-3} - \frac{3x^2+2}{x^3+2x} - \frac{1}{x-1}$$

multiply by $y(x)$

$$y'(x) = y(x)\left[\frac{2}{x} + \frac{1}{x-3} - \frac{3x^2+2}{x^3+2x} - \frac{1}{x-1}\right]$$

$$y'(x) = \boxed{\frac{x^2(x-3)}{(x^3+2x)(x-1)}\left(\frac{2}{x} + \frac{1}{x-3} - \frac{3x^2+2}{x^3+2x} - \frac{1}{x-1}\right)}$$

Ex: Use logarithmic diff to find $f'(x)$, if

$$f(x) = \frac{\sqrt[3]{x+1}}{(1-3x)^4}$$

$$\ln f(x) = \ln\left(\frac{\sqrt[3]{x+1}}{(1-3x)^4}\right)$$

$$= \ln\left[(x+1)^{1/3}\right] - \ln\left[(1-3x)^4\right]$$

$$\ln f(x) = \frac{1}{3}\ln(x+1) - 4\ln(1-3x)$$

$\downarrow$ diff.

$$\frac{1}{f(x)} \cdot f'(x) = \frac{1}{3} \cdot \frac{1}{x+1} - 4 \cdot \frac{-3}{1-3x}$$

$$f'(x) = f(x)\left[\frac{1}{3(x+1)} + \frac{12}{1-3x}\right]$$

$$\boxed{f'(x) = \frac{\sqrt[3]{x+1}}{(1-3x)^4}\left[\frac{1}{3(x+1)} + \frac{12}{1-3x}\right]}$$

Section 4.4

Ex: Sketch $f(x) = x^2 - 8\ln x$

Domain: $(0,\infty)$

$$f'(x) = 2x - \frac{8}{x} = 0$$

$$\frac{2x^2}{x} - \frac{8}{x} = 0$$

$x = 0$

$2x^2 - 8 = 0$

$x^2 = 4$

$$\frac{2x^2-8}{x}=0 \qquad \underline{x=\pm 2}$$

$(0,2)$ $(2,\infty)$

0 2

f'	$(0,2)$	$(2,\infty)$
$2(x^2-4)$	$-$	$+$
x	$+$	$+$
	$-$	$+$

$\boxed{x=2 \text{ rel. min}}$

$$f'(x)=2x-\frac{8}{x}=2x-8x^{-1}$$

$$f''(x)=2+8x^{-2}=\frac{2x^2}{x^2}+\frac{8}{x^2}=\frac{2x^2+8}{x^2}$$

$2x^2+8=0$

$x^2=-4$

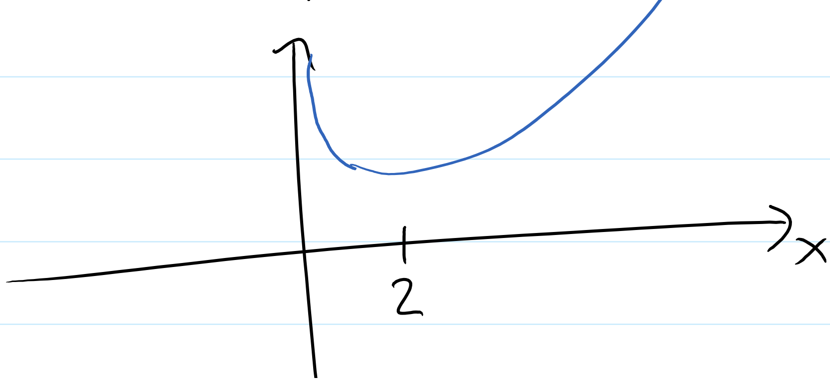
$\times$

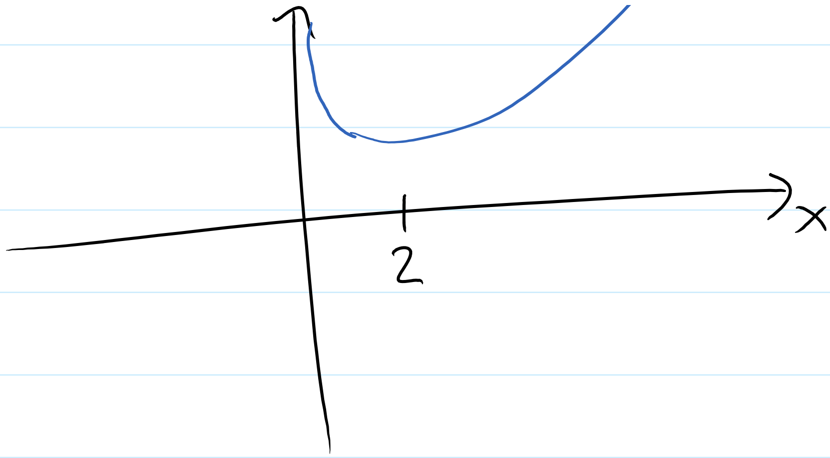
$x^2=0$

$x=0$

f''	$(0,\infty)$
$\frac{2x^2+8}{x^2}$	$+$

$f(x)=x^2-8\ln x$





Ex: The total number of hamburgers sold by a fast-food chain is growing exponentially. If 4 billion had been sold by 2005 (0,4) and 12 billion had been sold by 2010, (5,12) how many will have been sold by 2015? (10, ?)

$$H(x) = A \cdot e^{Bx}$$

Using (0,4): $H(0) = 4$

$$\left.\begin{aligned} A \cdot e^{B\cdot 0} &= 4 \\ A \cdot 1 &= 4 \end{aligned}\right\} \quad H(x) = 4e^{Bx}$$

Using (5,12): $H(5) = 12$

$$4e^{B\cdot 5}=12$$
$$e^{5B}=3$$
$$5B\cdot \ln e=\ln 3$$
$$5B=\ln 3$$

$$B=\frac{\ln 3}{5}$$

$$H(x)=4e^{\frac{\ln 3}{5}x}$$
$$=4\left(e^{\ln 3}\right)^{\frac{x}{5}}$$
$$=4\cdot 3^{x/5}$$

$$H(15)=4\cdot 3^{15/5}=4\cdot 3^{3}=4\cdot 27$$
$$=\boxed{108}$$

Formulas: elasticity, Future value