Review:

40

$q^2 + 5pq = 24$

a) Find elasticity
b) $E(2)$?

$$E(p) = -p\frac{q'(p)}{q(p} = -p\frac{q'}{q}$$

Need to find the function $q(p) = ?$

Solve: $q^2 + 5pq = 24$ for q.

$$q^2 + 5pq - 24 = 0$$

if: $q + 5pq = 24$
$q(1+5p) = 24$
$q = \frac{24}{1+5p}$

$$q = \frac{-5p \pm \sqrt{(5p)^2 + 4 \cdot 24}}{2}$$

$$= \frac{-5p \pm \sqrt{25p^2 + 96}}{2}$$

$$q(p) = \frac{-5}{2}p + \frac{1}{2}(25p^2 + 96)^{\frac{1}{2}}$$

$$q(p) = \frac{-5}{2}p + \frac{1}{2}\left(25p^2+96\right)^{\frac{1}{2}}$$

$$q'(p) = -\frac{5}{2} + \frac{1}{2}\cdot\frac{1}{2}\left(25p^2+96\right)^{-\frac{1}{2}}\cdot\left(25p^2+96\right)'$$

$$= -\frac{5}{2} + \frac{1}{\cancel{4}_{2}}\left(25p^2+96\right)^{-\frac{1}{2}}\left(\cancel{50}^{25}p\right)$$

$$= -\frac{5}{2} + \frac{25p}{2\sqrt{25p^2+96}}$$

$$E(p) = -\frac{p\left(-\frac{5}{2}+\frac{25p}{2\sqrt{25p^2+96}}\right)}{-\frac{5}{2}p+\frac{1}{2}\sqrt{25p^2+96}} = \frac{5p}{2\boxed{q}+5p}$$

$$\boxed{q} = \sqrt{25p^2+96}$$

b) Find $E(2)$.

$$E(2) = \frac{-2\left(\frac{-5}{2}+\frac{25\cdot\cancel{2}}{\cancel{2}\sqrt{25\cdot4+96}}\right)}{-\frac{5}{\cancel{2}}\cdot\cancel{2}+\frac{1}{2}\sqrt{25\cdot4+96}}$$

$$= \frac{5-\frac{50}{\sqrt{100+96}}}{-5+\frac{1}{2}\sqrt{196}} = \frac{5-\frac{50}{14}}{-5+\frac{1}{2}\cdot14} = \frac{5-\frac{25}{7}}{-5+7}$$

$$= \frac{10/7}{2} = \boxed{\frac{5}{7}}$$

$\frac{5}{7} < 1 \Rightarrow$ demand is inelastic

43

$C(n) = 3n^2 + 300n + 700$ ... cost of building with n floors.

$Ave(n) = \boxed{\frac{C(n)}{n}}$ ← minimize

$$Ave(n) = \frac{3n^2 + 300n + 700}{n} = 3n + 300 + 700n^{-1}$$

$$A'(n) = 3 + 0 - 700n^{-2} = 0$$

$$3 - 700n^{-2} = 0$$

$$n^{-2}(3n^2 - 700) = 0$$

$n^{-2} = 0$

$\boxed{n = 0}$

$3n^2 - 700 = 0$

$n^2 = \frac{700}{3}$

$n = \pm\sqrt{\frac{700}{3}} \approx \pm 15.275$

$n = 0, \sqrt{\frac{700}{3}} \approx 15.275$

$A''(n) = 1400n^{-3} > 0$ if $n > 0$ so

$$n = \sqrt{\frac{700}{3}} \approx 15.275 \text{ is a relative min.}$$

$$n = \lfloor 15.275 \rfloor = \boxed{15}$$

(47)

invest 9000, in 4 years we have 11,000. Find the interest rate if the interest is compounded

a) Quarterly.

$$B(t) = P\left(1 + \frac{r}{k}\right)^{kt}$$

$$P = 9000;\ B(4) = 11000;\ t = 4;\ k = 4$$

$$\frac{11000}{9000} = \frac{9000\left(1 + \frac{r}{4}\right)^{4 \cdot 4}}{9000}$$

$$\frac{11}{9} = \left(1 + \frac{r}{4}\right)^{16} \qquad / \ln$$

$$\ln 11/9 = \ln\left(\left(1 + r/4\right)^{16}\right)$$

$$\frac{\ln 11/9}{16} = \frac{16}{16} \cdot \ln\left(1 + r/4\right)$$

$$\frac{1}{16} \ln(11/9) = \ln\left(1 + r/4\right)$$

$\sqrt[16]{\ }$

$$\frac{1}{16}\ln(11/9) = \ln(1+r/4)$$

$$\ln\left(\left(\frac{11}{9}\right)^{1/16}\right) = \ln(1+r/4) \qquad /e^{\cdots}$$

$$\left(\frac{11}{9}\right)^{1/16} = 1+r/4$$

$$\frac{r}{4} = \sqrt[16]{\left(\frac{11}{9}\right)} - 1$$

$$r = \boxed{4\sqrt[16]{\frac{11}{9}} - 4} \approx 0.0505$$

b) continuously

$$11000 = 9000e^{r\cdot 4}$$

$$11/9 = e^{4r} \qquad /\ln$$

$$\ln 11/9 = 4r\cdot\ln e$$

$$\ln 11/9 = 4r$$

$$r = \boxed{\frac{1}{4}\ln 11/9} \approx 0.050167$$

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56 $f(x) = x^6\,5^{x^5}$

$$f'(x) = 5^{x^5}\cdot 6x^5 + x^6\cdot 5^{x^5}\cdot\ln(5)\cdot(x^5)'$$
$$= 5^{x^5}\cdot 6x^5 + x^6\cdot 5^{x^5}\cdot\ln 5\cdot 5x^4$$

$$= \boxed{6x^5\, 5^{x^5} + 5x^{10} \ln 5 \cdot 5^{x^5}}$$

$$= \boxed{5^{x^5} x^5 \left(6 + 5x^5 \ln 5\right)}$$

(11) $f(t) = \dfrac{t}{t^2+11}$, find crit. num. and rel. min/max

$$f'(t) = \frac{(t^2+11)\cdot 1 - t\cdot 2t}{(t^2+11)^2} = \frac{t^2+11-2t^2}{(t^2+11)^2} = \frac{11-t^2}{(t^2+11)^2}$$

$$11 - t^2 = 0 \qquad\qquad t^2+11 = 0$$

$$11 = t^2 \qquad\qquad t^2 = -11$$

$$t = \pm\sqrt{11} \qquad\qquad \text{no sol.}$$

$\boxed{\text{critical num: } t = \pm\sqrt{11}}$

$-\sqrt{11}$ $\quad$ $\sqrt{11}$

f'	$(-\infty, -\sqrt{11})$	$(-\sqrt{11}, \sqrt{11})$	$(\sqrt{11}, \infty)$
$11-t^2$	$-$ $\quad -4$	$+$ $\quad 0$	$-$ $\quad 4$
$(t^2+11)^2$	$+$	$+$	$+$
	$-$ ↘	$+$ ↗	$-$ ↘

$$\boxed{\begin{array}{l} t = -\sqrt{11} \text{ rel. min} \\ t = \sqrt{11} \text{ rel. max} \end{array}}$$

(45) $P = 16000$, $t = 8$, $r = 0.07$, contin. interest

$$B(t) = P e^{rt}$$

$$B(8) = 16000 \cdot e^{0.07 \cdot 8} = \boxed{16000\, e^{0.56}}$$

(46) $t = 3$, $B(3) = 29000$, $r = 0.06$, $k = 4$

$$29000 = P\left(1 + \frac{0.06}{4}\right)^{4 \cdot 3}$$

$$29000 = P(1 + 0.015)^{12}$$

$$P = \boxed{\frac{29000}{1.015^{12}}}$$

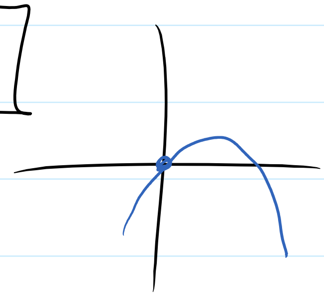
HW 3.4, #14

$$p(q) = 33 - 3q$$

$$C(q) = 2q^2 + 3q + 60$$

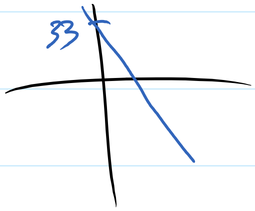
$R(q) = ?$ (revenue)

$$R(q) = p(q) \cdot q = (33 - 3q)q = \boxed{33q - 3q^2}$$



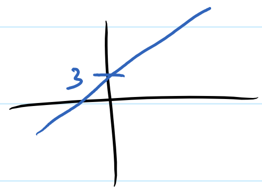
$R'(q) = ?$ (marginal revenue)

$$R'(q) = \boxed{33 - 6q}$$



$C'(q) = ?$

$$C'(q) = \boxed{4q + 3}$$



$P(q) = ?$ (profit) maximize

$$P(q) = R(q) - C(q)$$
$$= 33q - 3q^2 - (2q^2 + 3q + 60)$$
$$= 33q - 3q^2 - 2q^2 - 3q - 60$$
$$= \boxed{-5q^2 + 30q - 60}$$

$$P'(q) = -10q + 30 = 0$$
$$-10q = -30$$
$$q = +\frac{30}{10} = \boxed{3}$$

$P''(q) = -10$; $P''(3) = -10 < 0 \Rightarrow q = 3$ is a rel. max