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HW for 5.1 posted.

function	derivative
x^5	$5x^4$
x^3	$3x^2$
x^2	$2x$
$\frac{1}{3}x^3$	x^2
$\frac{1}{5}x^5$	x^4
x^{-3}	$-3x^{-4}$
$\frac{1}{-3}x^{-3}$	x^{-4}
$\frac{1}{-1}x^{-1}$	x^{-2}
$\ln x$	$x^{-1} = \frac{1}{x}$
$x^2(x-2)^4$	$(x-2)^4 \cdot 2x + x^2 \cdot 4(x-2)^3 \cdot 1$
	$2x(x-2)^4 + 4x^2(x-2)^3$
$x\ln x - x$	$\ln x \cdot 1 + x \cdot \frac{1}{x} - 1 = \ln x + 1 - 1 = \ln x$

$x\ln x - x \qquad \ln x$

Def: A function $F(x)$ is said to be an antiderivative of $f(x)$ if

$$F'(x) = f(x) \quad \text{or} \quad \frac{d}{dx} F(x) = f(x)$$

for every x in the domain of $f(x)$.
The process of finding antiderivatives is called antidifferentiation or indefinite integration.

Ex: Verify that $F(x) = \frac{1}{3}x^3 + 5x + 2$ is an antiderivative of $f(x) = x^2 + 5$.

$$\frac{d}{dx} F(x) = \left(\tfrac{1}{3}x^3 + 5x + 2\right)' = \tfrac{1}{3} \cdot 3x^2 + 5 = x^2 + 5 \checkmark$$

Ex: Find two different antiderivatives of $f(x) = x^2 + x$.

$$\left.\begin{aligned} F_1(x) &= \tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 \\ F_2(x) &= \tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + \tfrac{1}{2} \\ F_3(x) &= \tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 - 4 \end{aligned}\right\} \text{ we can add any number since } (c)' = 0$$

Thm: (Fundamental Property of Antiderivatives)
If $F(x)$ is an antiderivative of $f(x)$, then any other antiderivative of $f(x)$ has the form $G(x) = F(x) + C$ for some constant C.

Rules (for finding antiderivatives)

- Constant rule: $f(x) = a \rightarrow F(x) = ax + C$, where C is a const.

- Power rule: $f(x) = x^n \rightarrow F(x) = \frac{1}{n+1}x^{n+1} + C$, $n \neq -1$

- logarithmic rule: $f(x) = \frac{1}{x} \rightarrow F(x) = \ln|x| + C$

- Exponential rule: $f(x) = e^x \rightarrow F(x) = e^x + C$

  - $f(x) = e^{kx} \rightarrow F(x) = \frac{1}{k}e^{kx} + C$

Notation:

$$\int f(x)\, dx = F(x)$$

$\int x^2\, dx =$ "find the antiderivative of x^2" $= \frac{1}{3}x^3 + C$

The general antiderivative of a function always has $+C$.

$$\int x^n\,dx = \begin{cases} \frac{1}{n+1}x^{n+1}, & \text{if } n \neq -1 \\ \ln|x|, & \text{if } n = -1 \end{cases}$$

Evaluate:

- $\int 3\,dx = 3x + C$
- $\int x^{17}\,dx = \frac{1}{18}x^{18} + C$
- $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = 2x^{1/2} + C$
- $\int e^{-3x}\,dx = \frac{1}{-3}e^{-3x} + C$

Rules

- Constant multiple:

$$\int k\cdot f(x)\,dx = k\cdot\int f(x)\,dx, \quad \text{for constant } k$$

- Sum/difference rule:

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Ex: • $\int (2x^5 + 8x^3 - 3x^2 + 5)\,dx$

$$= 2\cdot\frac{1}{6}x^6 + 8\frac{1}{4}x^4 - 3\frac{1}{3}x^3 + 5x + C$$

$$= \boxed{\frac{1}{3}x^6 + 2x^4 - x^3 + 5x + C}$$

- $$\int \frac{x^3+2x-7}{x}\,dx = \int x^2 + 2 - \frac{7}{x}\,dx$$

$$= \frac{1}{3}x^3 + 2x - 7\int \frac{1}{x}\,dx = \boxed{\frac{1}{3}x^3 + 2x - 7\ln|x| + C}$$

- $$\int 3e^{-5t} + \sqrt{t}\,dt = \int 3e^{-5t} + t^{1/2}\,dt$$

$$= 3\frac{1}{-5}e^{-5t} + \frac{2}{3}t^{3/2} + C$$

$$= \boxed{-\frac{3}{5}e^{-5t} + \frac{2}{3}t^{3/2} + C}$$

Ex:

... marginal cost of certain product $3q^2 - 60q + 400$ dollars per unit. The total cost of producing two units is \$900. Find the total cost function and the total cost of producing 5 units.

$$\begin{cases} C'(q) = 3q^2 - 60q + 400 \\ C(2) = 900 \end{cases}$$ (initial value problem)

$$C(q) = \int c'(q)\,dq = \int 3q^2 - 60q + 400\,dq$$

$$= 3\tfrac{1}{3}q^3 - 60\tfrac{1}{2}q^2 + 400q + C$$

$$= q^3 - 30q^2 + 400q + C$$

use $C(2) = 900$ to find C.

$$C(2) = 900$$

$$2^3 - 30\cdot 2^2 + 400\cdot 2 + C = 900$$

$$8 - 120 + 800 + C = 900$$

$$C = 900 - 800 + 120 - 8$$

$$C = 220 - 8$$

$$\boxed{C = 212}$$

$$\boxed{C(q) = q^3 - 30q^2 + 400q + 212}$$

$$C(5) = 125 - 750 + 2000 + 212 = \ldots = \boxed{1587}$$

Ex: Solve:

$$\frac{dy}{dx} = \frac{2}{x} - \frac{1}{x^2}$$, where $y = -1$ when $x = 1$.

Find $y(x)$.

$$\begin{cases} y'(x) = \frac{2}{x} - \frac{1}{x^2} \\ y(1) = -1 \end{cases}$$

$$y(x) = \int \frac{2}{x} - x^{-2}\, dx = 2\ln|x| - \frac{1}{-1}x^{-1} + C$$

$$= 2\ln|x| + \frac{1}{x} + C$$

$$y(1) = -1$$
$$2 \cdot \ln|1| + \frac{1}{1} + C = -1$$

$$0 + 1 + C = -1$$
$$C = -2$$

$$\boxed{y(x) = 2\ln|x| + \frac{1}{x} - 2}$$

Find :

- $\int \frac{1}{x}(x+1)^2\, dx = \int \frac{1}{x}(x^2+2x+1)\,dx$
  $= \int x + 2 + \frac{1}{x}\, dx = \boxed{\frac{1}{2}x^2 + 2x + \ln|x| + C}$

- $\int \frac{e^x}{2} + x\sqrt{x}\, dx = \int \frac{1}{2}e^x + x^{3/2}\, dx$
  $= \boxed{\frac{1}{2}e^x + \frac{2}{5}x^{5/2} + C}$