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Tuesday, October 31, 2017 2:10 PM

Section 5.1

- $\int x^4\,dx = \frac{1}{5}x^5 + C$

- $\int \frac{x^2-2x+3}{x}\,dx = \int \frac{x^2}{x} - \frac{2x}{x} + \frac{3}{x}\,dx$

  $= \int x - 2 + \frac{3}{x}\,dx = \boxed{\frac{1}{2}x^2 - 2x + 3\ln|x| + C}$

- $\int e^{-2x} - \sqrt{x}\,dx = \int e^{-2x} - x^{1/2}\,dx$

  $= \frac{1}{-2}e^{-2x} - \frac{2}{3}x^{3/2} + C$

  $= \boxed{-\frac{1}{2}e^{-2x} - \frac{2}{3}x^{3/2} + C}$

Section 5.2 - Integration by substitution

What is the "inverse" of chain rule?

f	f'
$(2x-1)^5$	$5(2x-1)^4 \cdot 2$
$(x^4-x)^7$	$7(x^4-x)^6 \cdot (4x^3-1)$
$(4x^{-2}+3)^{-3}$	$-3(4x^{-2}+3)^{-4} \cdot (-8x^{-3})$
$\ln(x^3-x)$	$\dfrac{3x^2-1}{x^3-x}$

f	f'
$\frac{1}{7}(x^4-x)^7$	$(x^4-x)^6(4x^3-1)$
$\frac{1}{2}e^{x^2}$	$e^{x^2} \cdot x$
$\ln(2x^4+3x)$	$\dfrac{8x^3+3}{2x^4+3x}$

The chain rule: (for derivatives)

$$\frac{d}{dx} f(u(x)) = \frac{d}{dx} f(u) = \frac{d}{du} f(u) \cdot \frac{d}{dx} u(x)$$

$$(f(u(x)))' = (f(u))' = f'(u) \cdot u'(x)$$

So

$$\int f'(u) \cdot u'(x)\, dx = f(u(x)) + C.$$

What is the differential of $u(x)$?

$$du = u'(x)\, dx$$

So...

$$\int f'(u)\cdot u'(x)\,dx = \int f'(u)\,du$$

Ex:

$$\int 7(x^4-x)^6\cdot(4x^3-1)\,dx =$$

let $u = x^4 - x$

$du = (4x^3-1)\,dx$

$$= \int 7(\ u\ )^6\ du = \int 7u^6\,du$$

$$= 7\int u^6\,du = 7\tfrac{1}{7}u^7 + C = u^7 + C$$

(back substitution)

$$= \boxed{(x^4-x)^7 + C}$$

Ex: Find $\int \sqrt{2x+7}\,dx = \int (2x+7)^{1/2}\,dx$

$u = 2x+7$

$\frac{du}{2} = \frac{2\,dx}{2}$

$\frac{1}{2}du = dx$

$$= \int (u)^{1/2}\,\tfrac{1}{2}\,du$$

$$= \tfrac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} u^{3/2} + C = \boxed{\frac{1}{3}(2x+7)^{3/2} + C}$$

$$\frac{d}{dx}\left[\frac{1}{3}(2x+7)^{3/2} + C\right] = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}}(2x+7)^{1/2} \cdot \cancel{2} + 0$$

$$= (2x+7)^{1/2} \quad \checkmark$$

• $\int 8x(4x^2-3)^5\,dx = \int u^5\,du$

$u = 4x^2 - 3$

$du = 8x\,dx$

$$= \frac{1}{6}u^6 + C$$

$$= \boxed{\frac{1}{6}(4x^2-3)^6 + C}$$

• $\int x^3 e^{x^4+2}\,dx$

$\cancel{u = x^3}$

$\cancel{du = 3x^2\,dx}$

$\cancel{\frac{du}{3} = \frac{3x^2}{3}\,dx}$

$\cancel{\frac{1}{3}\,du = x^2\,dx}$

$u = x^4 + 2$

$du = 4x^3\,dx$

$\frac{du}{4} = \frac{4x^3}{4}\,dx$

$\frac{1}{4}\,du = x^3\,dx$

$$= \int e^u \frac{1}{4}\,du = \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C$$

$$= \boxed{\frac{1}{4}e^{x^4+2} + C}$$

$$= \boxed{\frac{1}{4} e^{x^4+2} + C}$$

- $\displaystyle\int \frac{x}{x-1}\,dx$ | $u = x-1$, $du = dx$ | $\displaystyle = \int \frac{u+1}{u}\,du$

$x = ?$

$u+1 = x$

$$= \int 1 + \frac{1}{u}\,du = u + \ln|u| + C = \boxed{x-1+\ln|x-1|+C}$$

- $\displaystyle\int \frac{3x+6}{\sqrt{2x^2+8x+3}}\,dx$

$u = \sqrt{2x^2+8x+3}$

$u = (2x^2+8x+3)^{1/2}$

$du = \frac{1}{2}(2x^2+8x+3)^{-1/2}(4x+8)\,dx$

$u = 2x^2+8x+3$

$du = (4x+8)\,dx$

$\frac{1}{4}du = (x+2)\,dx$

$$\int \frac{3x+6}{\sqrt{2x^2+8x+3}}\,dx$$

$$= \int \frac{3(x+2)}{}\,dx$$

$du = (4x+8)\,dx$

$\frac{1}{4}du = (x+2)\,dx$

$$= \int \frac{3(x+2)}{\sqrt{2x^2+8x+3}}\,dx$$

$$= \int \frac{3}{\sqrt{u}}\,\frac{1}{4}du = \frac{3}{4}\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{3}{4}\int u^{-1/2}\,du = \frac{3}{4}\,\frac{2}{1}\,u^{1/2} + C = \frac{3}{2}\sqrt{u} + C$$

$$= \boxed{\frac{3}{2}\sqrt{2x^2+8x+3} + C}$$

• $$\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$$

$u = e^x - e^{-x}$

$du = e^x - e^{-x}\cdot(-1)\,dx$

$du = (e^x + e^{-x})\,dx$

$$= \int \frac{1}{u}\,du = \ln|u| + C$$

$$= \boxed{\ln|e^x - e^{-x}| + C}$$