11/02

Thursday, November 2, 2017 1:49 PM

HW 16 -5.1 due Sunday 11/04.

Ex: Marginal cost: $C'(x) = 5x - 2$,
given that: $C(2) = 0$ Find $C(x)$.

$$C(x) = \int 5x - 2\,dx = 5\tfrac{1}{2}x^2 - 2x + c$$

$$C(x) = \frac{5}{2}x^2 - 2x + c$$

↳ some constant

$$C(2) = 0$$
$$\frac{5}{2}\cdot(2)^2 - 2\cdot 2 + c = 0$$
$$\frac{5}{\cancel{2}}\cdot \cancel{4}^{2} - 4 + c = 0$$

$$10 - 4 + c = 0$$
$$\boxed{c = -6}$$

$$\boxed{C(x) = \frac{5}{2}x^2 - 2x - 6}$$

Section 5.2

Ex:

$$\int \frac{(\ln x)^2}{x}\,dx = \int \frac{u^2}{1}\,du = \int u^2\,du$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \frac{1}{3}u^3 + C = \boxed{\frac{1}{3}(\ln x)^3 + C}$$

Ex:

$$\int \frac{x^2+3x+5}{x+1}\,dx = \int \frac{(u-1)^2+3(u-1)+5}{u}\,du$$

$u = x+1 \longrightarrow x = u-1$

$du = dx \qquad x^2 = (u-1)^2$

Can we simplify $\frac{x^2+3x+5}{x+1}$?

long division:

$$\begin{array}{r} x+2 \\ x+1 \overline{)\, x^2+3x+5} \\ -(x^2+x) \\ \hline 2x+5 \\ -(2x+2) \\ \hline 3 \end{array}$$

$$\boxed{\frac{x^2+3x+5}{x+1} = x+2+\frac{3}{x+1}}$$

$$\int \frac{x^2+3x+5}{x+1}\,dx = \int x+2+\frac{3}{x+1}\,dx$$

$$= \int x+2\,dx + 3\int \frac{1}{x+1}\,dx$$

$u = x+1$

$du = dx$

$u = x+1$
$du = dx$

$= \frac{1}{2}x^2 + 2x + 3\int \frac{1}{u}\,du$

$= \frac{1}{2}x^2 + 2x + 3\ln|u|$

$= \boxed{\frac{1}{2}x^2 + 2x + 3\ln|x+1| + C}$

Ex: $\int x^4 e^{x^4+2}\,dx$

$u = x^4+2 \longrightarrow x^4 = u-2$
$x = \sqrt[4]{u-2}$

$\frac{du}{4} = \frac{4x^3}{4}\,dx$

$\frac{1}{4}\,du = x^3\,dx$

$= \int x\,x^3 e^{x^4+2}\,dx = \int \sqrt[4]{u-2}\,e^u \frac{1}{4}\,du$

$= \underbrace{\frac{1}{4}\int (u-2)^{\frac{1}{4}} e^u\,du}_{\text{not solvable}}$

We cannot use the substitution method to solve the integral.

Ex: The price p (dollars) of each unit of a commodity is estimated to be changing at the rate unit of a commodity is estimated to be changing at the rate

$$\frac{dp}{dx} = \frac{-135x}{\sqrt{9+x^2}}$$

where x ... hundred of units.
Suppose 400 units $(x=4)$ are demanded when the price is \$30.

a) Find the dem. function $p(x)$.

$$\begin{cases} p'(x) = \dfrac{-135x}{(9+x^2)^{1/2}} \\ p(4) = 30 \end{cases}$$

$$p(x) = ?$$

$$\int p'(x)\,dx = \int \frac{-135x}{(9+x^2)^{1/2}}\,dx$$

$$u = 9+x^2$$
$$du = 2x\,dx$$
$$\tfrac{1}{2}du = x\,dx$$

$$= \int \frac{-135}{(u)^{1/2}}\,\frac{1}{2}du = \frac{-135}{2}\int u^{-1/2}du$$

$$= \frac{-135}{\cancel{2}}\,\cancel{2}\,u^{1/2} + C = -135\sqrt{9+x^2} + C$$

$$= \frac{-135}{\cancel{2}} \cdot \frac{\cancel{2}}{1} u^{1/2} + C = \boxed{-135\sqrt{9+x^2} + C}$$

$$p(4) = 30$$

$$-135\sqrt{9+4^2} + C = 30$$

$$-135\sqrt{25} + C = 30$$

$$-135 \cdot 5 + C = 30$$

$$C = 30 + 675$$

$$C = 705$$

$$\boxed{p(x) = -135\sqrt{9+x^2} + 705}$$

b) At what price will no units be demanded?

$$p(x) = 0 \Longrightarrow$$ "price is zero, find the quantity"

$$-135\sqrt{9+x^2} + 705 = 0$$

$$\sqrt{9+x^2} = \frac{\cancel{705}^{141}}{\cancel{135}_{27}} = \frac{141}{27} = \frac{47}{9}$$

$$9+x^2 = \left(\frac{47}{9}\right)^2$$

$$x^2 = \left(\frac{47}{9}\right)^2 - 9$$

$$x = \left(\frac{47}{9}\right) - 9$$

$$x = \sqrt{\left(\frac{47}{9}\right)^2 - 9} \approx 4.27$$

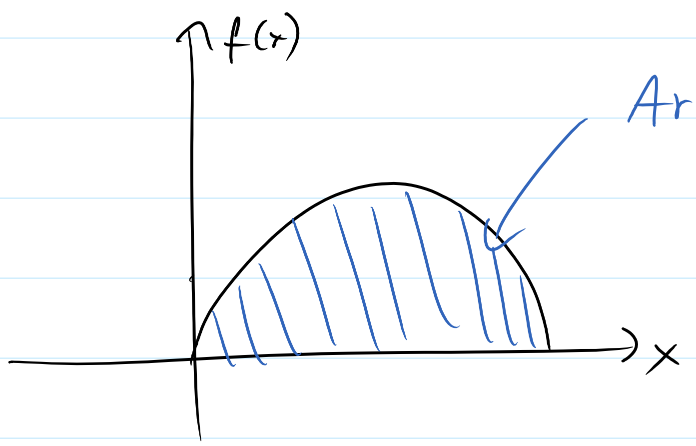
need to solve: $p(0)$

$$p(0) = -135\sqrt{9+0^2} + 705$$

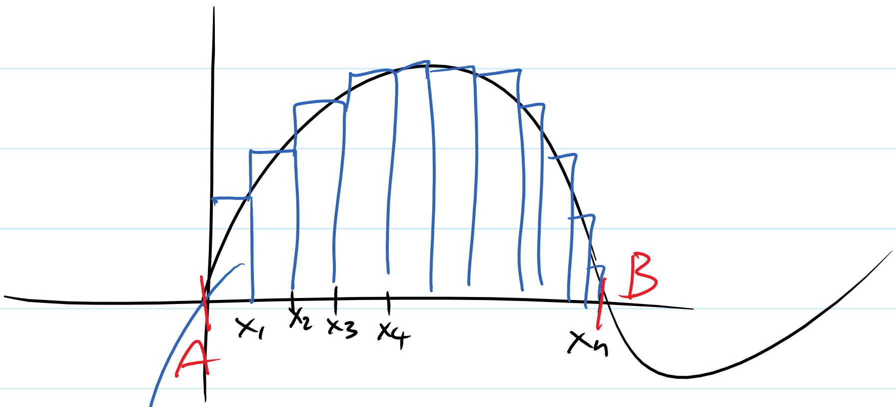
$$= -135\sqrt{9} + 705 = -405 + 705$$

$$= \boxed{300}$$

5.3 - Definite integrals (Area)



We can estimate the area under a function using rectangls



$$\text{Area} \simeq \underbrace{\begin{pmatrix}\text{width} \\ \text{of} \\ \text{each} \\ \text{rect}\end{pmatrix}}_{= \Delta x} \cdot \left[f(x_1) + f(x_2) + f(x_3) + \ldots + f(x_n) \right]$$

$$\begin{pmatrix}\text{Area under} \\ \text{a curve}\end{pmatrix} \simeq \Delta x \cdot \left(f(x_1) + f(x_2) + \ldots + f(x_n) \right)$$

$$\text{Area} = \lim_{\Delta x \to 0} \Delta x \cdot \sum_{i=1}^{n} f(x_i)$$

$$= \quad (\Delta x) = dx$$

$$\sum = \int$$

$$= \int_A^B f(x)\, dx$$

Def: Area as a Definite integral

Def: Area as a Definite integral

If $f(x)$ is a continuous function and $f(x) \geq 0$ on the interval $a \leq x \leq b$, then the region R under the curve $y = f(x)$ over the interval $[a,b]$ has area A given by the definite integral

$$A = \int_a^b f(x)\, dx$$

Thm: (The Fundamental Thm of Calculus)

If the function $f(x)$ is continuous on $[a,b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a),$$

where $F(x)$ is any antiderivative of $f(x)$ on $[a,b]$.

Ex: Use the FTC (Fundam. thm. of Calculus) to find the area of the region under the line $y = 2x+1$ over the the interval $[1,3]$.

$$\int_1^3 2x+1\, dx = F(3) - F(1)$$
$$= (3^2+3) - (1^2+1)$$

$$= (3^2+3) - (1^2+1)$$

$$\int 2x+1\,dx = x^2+x+C$$

$$F(x) = x^2+x \qquad = 12 - 2 = \boxed{10}$$

$$\int_1^3 2x+1\,dx = x^2+x \Big|_1^3 = (3^2+3)-(1^2+1) = \boxed{10}$$

Ex: Evaluate:

$$\int_0^1 e^{-x} + \sqrt{x}\,dx = -e^{-x} + \frac{2}{3}x^{3/2}\Big|_0^1$$

$$= \left(-e^{-1} + \frac{2}{3}(1)\right) - \left(-e^{-0} + \frac{2}{3}0^{3/2}\right)$$

$$= -\frac{1}{e} + \frac{2}{3} - (-1 + 0)$$

$$= -\frac{1}{e} + \frac{2}{3} + 1 = \boxed{\frac{5}{3} - \frac{1}{e}}$$