11/07

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Section 5.3

Evaluate:

$$\int_1^4 \frac{1}{x} - x^2 \, dx = \ln|x| - \frac{1}{3}x^3 \Big|_1^4$$

$$= \ln|4| - \frac{1}{3}4^3 - \left(\ln|1| - \frac{1}{3}1^3\right)$$

$$= \ln 4 - \frac{64}{3} - 0 + \frac{1}{3} = \ln 4 - \frac{63}{3}$$

$$= \boxed{-21 + \ln 4} = \boxed{\ln(4) - 21}$$

Rules for def. integrals

1) Constant multiple:

$$\int_a^b k f(x) \, dx = k \int_a^b f(x) \, dx$$

2) Sum / difference:

$$\int_a^b f(x) \pm g(x) \, dx = \int_a^b f(x) \, dx \pm \int_a^b g(x) \, dx$$

3)

- $\int_a^a f(x) \, dx = 0$

3)

- $\int_a^a f(x)\,dx = 0$
- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

4) Subdivision rule:

if $a < b < c$ are constants, then

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

<u>Ex:</u> given

$$\int_{-2}^{5} f(x)\,dx = 3,\quad \int_{-2}^{5} g(x)\,dx = -4,\quad \int_{3}^{5} f(x)\,dx = 7$$

find:

- $\int_{-2}^{5} 2f(x) - 3g(x)\,dx = \int_{-2}^{5} 2f(x)\,dx - \int_{-2}^{5} 3g(x)\,dx$

$$= 2\int_{-2}^{5} f(x)\,dx - 3\int_{-2}^{5} g(x)\,dx$$

$$= 2\cdot(3) - 3\cdot(-4) = 6 + 12 = \boxed{18}$$

- $\int_{-2}^{3} f(x)\,dx$

Subdiv. rule: $\int_{-2}^{5} f(x)\,dx = \int_{-2}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$

$$3 = \int_{-2}^{3} f(x)\,dx + 7$$
$$-7 \qquad\qquad -7$$

$$\boxed{-4 = \int_{-2}^{3} f(x)\,dx}$$

Evaluate: $\int_0^1 8x(x^2+1)^3\,dx$

$u = x^2+1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$x=0 \to u=1$
$x=1 \to u=1^2+1$
$=2$

$$= \frac{1}{2}\int_1^2 8\,u^3\,du$$

$$= \frac{1}{2}\cdot 8\int_1^2 u^3\,du = 4\cdot\frac{1}{4}u^4\Big|_1^2 = u^4\Big|_1^2$$

$$= 2^4 - 1^4 = 16-1 = \boxed{15}$$

no back substitution

$\boxed{\text{OR}}$ keep the original bounds and do back substitution.

$$= \frac{1}{2}\int_0^1 8u^3\,du = 4\int_0^1 u^3\,du = 4\,\frac{1}{4}\,u^4\Big|_0^1$$

$$= u^4\Big|_0^1 = (x^2+1)^4\Big|_0^1 = (1^2+1)^4 - (0^2+1)^4$$
$$= 2^4 - 1^4 = \boxed{15}$$

Evaluate: $\displaystyle\int_{\frac{1}{4}}^{2} \frac{\ln x}{x}\,dx = \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array}\right.$ $\quad x=\frac{1}{4} \to u = \ln\frac{1}{4}$, $\quad x = 2 \to u = \ln 2$

$$= \int_{\ln\frac{1}{4}}^{\ln 2} u\,du = \frac{1}{2}u^2 \Big|_{\ln\frac{1}{4}}^{\ln 2}$$

$$\boxed{\begin{aligned} \ln\tfrac{1}{4} &= \ln 2^{-2} \\ &= -2\ln 2 \end{aligned}}$$

$$= \frac{1}{2}u^2\Big|_{-2\ln 2}^{\ln 2}$$

$$= \frac{1}{2}(\ln 2)^2 - \left(\frac{1}{2}(-2\ln 2)^2\right)$$

$$= \frac{1}{2}(\ln 2)^2 - \frac{1}{2}\cdot 4\cdot(\ln 2)^2 = \frac{1}{2}(\ln 2)^2 - 2(\ln 2)^2$$

$$= (\ln 2)^2(0.5 - 2) = \boxed{-1.5(\ln 2)^2}$$

Net Change: If $Q'(x)$ is continuous on the interval $[a,b]$, then the net change in $Q(x)$ as x varies from $x=a$ to $x=b$ is given by

$$Q(b) - Q(a) = \int_a^b Q'(x)\,dx$$

Ex: Given marginal cost, $C'(q) = 3(q-4)^2$, by how much will the total cost increase if the level of production is raised from 6 units ($q=6$) to 10 units ($q=10$)

if the level of production is raised from 6 units ($q=6$) to 10 units ($q=10$)

$$C(10)-C(6)=\int_6^{10} 3(q-4)^2\,dq \quad \Bigg| \begin{array}{l} u=q-4 \\ du=dq \\ q=6 \to u=6-4=2 \\ q=10 \to u=10-4=6 \end{array}$$

$$=\int_2^6 3u^2\,du = 3\cdot\frac{1}{3}u^3\Big|_2^6 = u^3\Big|_2^6 = 6^3-2^3$$

$$=216-8=\boxed{208}$$

$$\begin{array}{r} 36 \\ \cdot 6 \\ \hline 216 \end{array}$$