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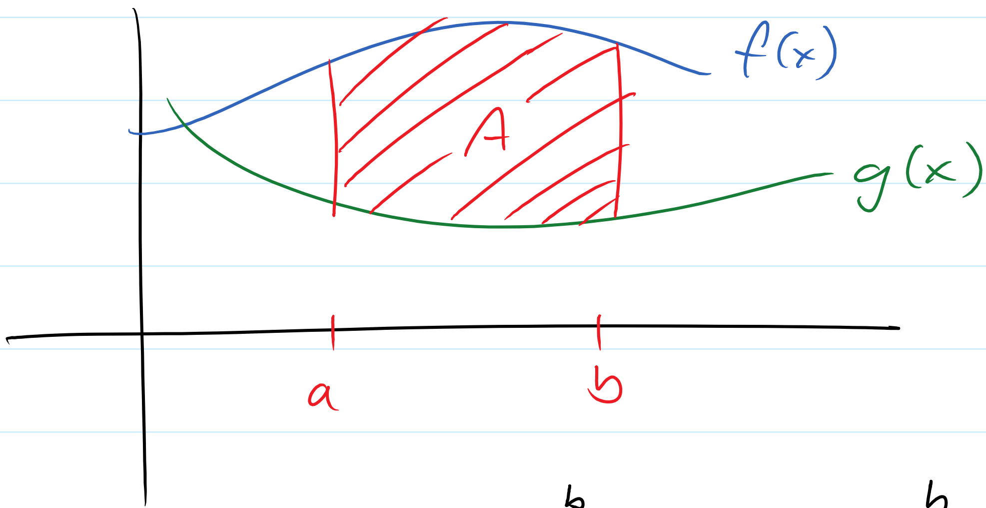
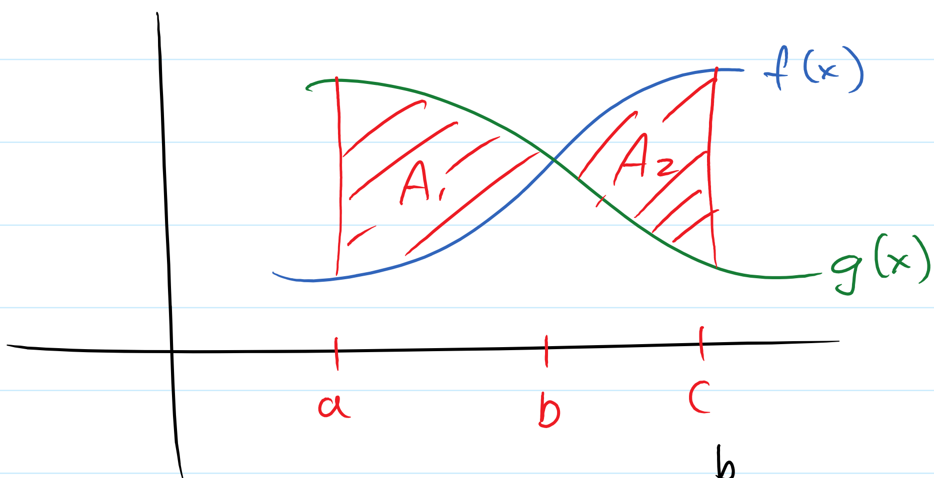
Tuesday, November 14, 2017 2:06 PM



area $A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$= \int_a^b f(x) - g(x)\,dx$



Area $A_1 + A_2 = \underbrace{\int_a^b g(x) - f(x)\,dx}_{A_1} + \underbrace{\int_b^c f(x) - g(x)\,dx}_{A_2}$

Definition:

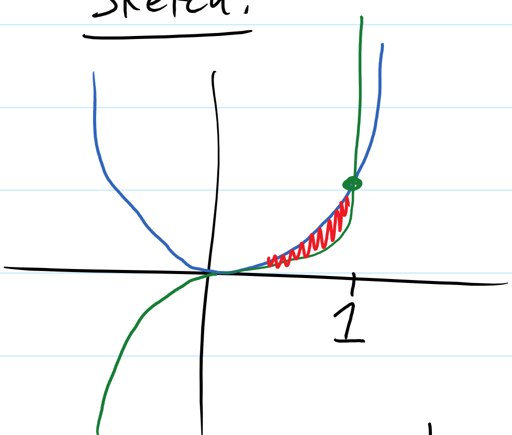
If f(x) and g(x) are continuous and

If $f(x)$ and $g(x)$ are continuous and $f(x) \geq g(x)$ on $[a,b]$, then the area A between the curves $y=f(x)$ and $y=g(x)$ over $[a,b]$ is

$$A = \int_a^b f(x) - g(x)\, dx$$

Ex: Find the area of the region enclosed by the curves $y=x^3$ and $y=x^2$.

sketch:



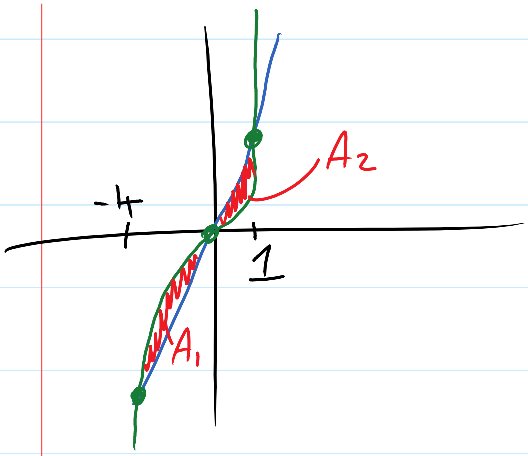
intersects:

$$x^2 = x^3$$
$$x^2 - x^3 = 0$$
$$x^2(1-x) = 0$$
$$x=0, \; x=1$$

$$A = \int_0^1 x^2 - x^3\, dx = \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 \Big|_0^1$$

$$= \tfrac{1}{3} - \tfrac{1}{4} - (0-0) = \tfrac{4}{12} - \tfrac{3}{12} = \boxed{\tfrac{1}{12}}$$

Ex: Find the area of the region enclosed by $y=4x$ and $y=x^3+3x^2$



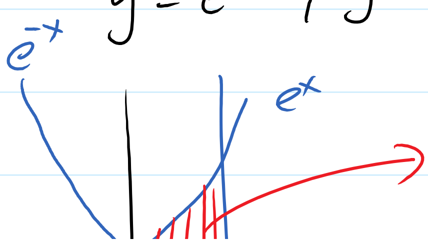
$$4x = x^3 + 3x^2$$
$$0 = x^3 + 3x^2 - 4x$$
$$0 = x(x^2 + 3x - 4)$$
$$0 = x(x-1)(x+4)$$
$$x = 0,\ 1,\ -4$$

$$A_1 = \int_{-4}^{0} x^3 + 3x^2 - 4x \, dx = \frac{1}{4}x^4 + x^3 - 2x^2 \Big|_{-4}^{0}$$
$$= \frac{1}{4}\cdot(0) + 0 - 0 - \left(\frac{1}{4}\cdot(-4)^4 + (-4)^3 - 2(-4)^2\right)$$
$$= -(\cancel{64} - \cancel{64} - 32) = 32$$

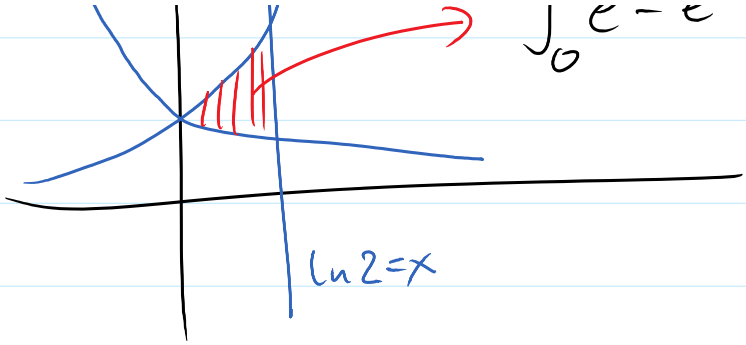
$$A_2 = \int_0^1 4x - (x^3 + 3x^2)\, dx = \int_0^1 4x - x^3 - 3x^2 \, dx$$
$$= 2x^2 - \frac{1}{4}x^4 - x^3 \Big|_0^1 = 2 - \frac{1}{4} - 1 - (0 - 0 - 0)$$
$$= 3/4$$

$$\text{Area} = 32 + 3/4 = \boxed{32.75} = \boxed{131/4}$$

Ex: Find the area of the region bounded by $y = e^x$, $y = e^{-x}$ and $x = \ln 2$



$$\int_0^{\ln 2} e^x - e^{-x}\, dx = e^x - \frac{1}{-1}e^{-x} \Big|_0^{\ln 2}$$



$$\int_0 e^x - e^{-x}\,dx = e^x + e^{-x}\Big|_0$$

$$= e^x + e^{-x}\Big|_0^{\ln 2}$$

$$= e^{\ln 2} + e^{-\ln 2} - (e^0 + e^0)$$

$$= 2 + \frac{1}{e^{\ln 2}} - (1+1)$$

$$= 2 + \frac{1}{2} - 2 = \boxed{1/2}$$

Ex: Suppose that t years from now, one investment will be generating profit at the rate $P_1'(t) = 50 + t^2$ hundred dollars per year, and the second investment .... at the rate $P_2'(t) = 200 + 5t$ 100s \$ per year.

a) For how long does the second investment rate exceeds the first one.

$$50 + t^2 = 200 + 5t$$

$$t^2 - 5t - 150 = 0$$

$$(t-15)(t+10) = 0$$

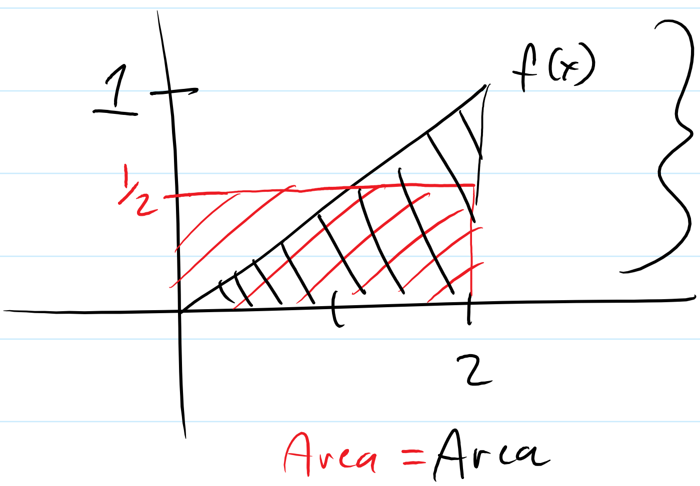
$$t = 15, \cancel{-10}$$

b) Compute the net excess profit for the time period $t=0$, $t=15$.

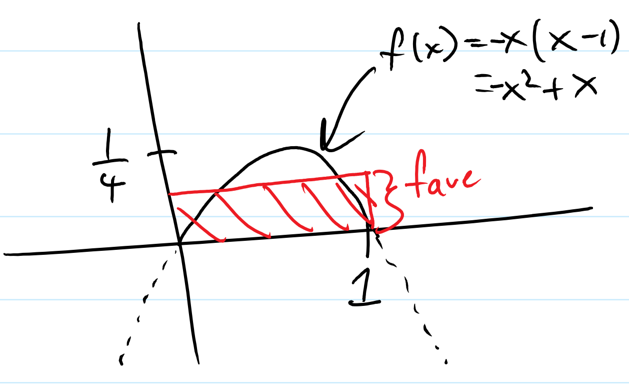
$$\int_0^{15} 200+5t-(50+t^2)\,dt = \int_0^{15} 150+5t-t^2\,dt$$

$$= 150t+\frac{5}{2}t^2-\frac{1}{3}t^3\Big|_0^{15} = 150\cdot 15+\frac{5}{2}\cdot 15^2-\frac{1}{3}15^3$$
$$-(0+0-0)$$
$$= \boxed{1687.5} \text{ hundred \$}$$

Average value



find the average value of $f(x)$ on $[0,2]$. $\boxed{\tfrac{1}{2}}$



find the average value of $f(x)$ on $[0,1]$.

$\square$ area $= f_{ave} \cdot (1-0)$

area $= \int_0^1 f(x)\, dx$

$$f_{ave} \cdot (1-0) = \int_0^1 f(x)\, dx$$

$$f_{ave} = \int_0^1 -x^2 + x \, dx = -\frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_0^1$$

$$= -\frac{1}{3} + \frac{1}{2} = \boxed{\frac{1}{6}}$$

Def: If $f(x)$ is continuous on $[a,b]$ then the average value of $f(x)$ on $[a,b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\, dx$$

Ex: Find the average value of

$$S(t) = \frac{750t}{\sqrt{4t^2+25}}$$

on $[0,6]$.

$$S_{ave} = \frac{1}{6-0}\int_0^6 \frac{750t}{\sqrt{4t^2+25}}\, dt = \left|\begin{array}{l} u = 4t^2+25 \\ du = 8t\ dt \\ \frac{1}{8}du = t\, dt \end{array}\right.$$

$$= \frac{1}{6}\cdot 750 \cdot \frac{1}{8}\int \frac{1}{\sqrt{u}}\, du = \frac{750}{48}\int u^{-1/2}\, du$$

$$= \frac{750}{\cancel{48}_{24}} \cdot \frac{\cancel{2}}{1} u^{1/2} \Big| = \frac{\cancel{750}^{375}}{\cancel{24}_{12}} \sqrt{4t^2+25} \Big|_0^6$$

$$= \frac{375}{12}\left(\sqrt{4\cdot 36+25} - \sqrt{0+25}\right)$$

$$= \frac{125}{4}\left(\sqrt{169} - \sqrt{25}\right) = \frac{125}{4}(13-5)$$

$$= \frac{125}{\cancel{4}} \cdot \cancel{8}^{2} = \boxed{250}$$