Online Office hour on Sunday at 6PM.

All online assignments are posted.

Exam on Tuesday : 5.1–5.5
(you will get formulas from 5.5)

Section 5.5

What is annuity?

Annuity is a fixed sum paid someone each year.

Future Value of an Income Stream

Suppose money is being transferred continuously into an account over a time perion $0 < t < T$ at a rate given by the function $f(t)$ (usually constant) and that the account earns interest at an annual rate r compounded continuously. Then the future value FV

continuously. Then the future value FV of income stream over the term T is given by

$$FV = \int_0^T f(t)\, e^{r(T-t)}\, dt = e^{rT} \int_0^T f(t)\, e^{-rt}\, dt$$

$$e^{rT} \cdot e^{-rt}$$

Ex: Use this formula to find out how much money you will have on a savings account if \$5000 is deposited per year with 5% interest after 15.

$r = 0.05$, $T = 15$, $f(t) = 5000$

$$FV = e^{0.05 \cdot 15} \int_0^{15} 5000\, e^{-0.05t}\, dt$$

$$= e^{.75} \cdot 5000 \int_0^{15} e^{-0.05t}\, dt$$

$$= 5000\, e^{0.75} \frac{1}{-0.05} e^{-0.05t} \Big|_0^{15}$$

$$= \frac{5000}{-0.05} e^{0.75} \left(e^{-0.05 \cdot 15} - e^{0} \right)$$

$$= \frac{5000}{-0.05} e^{0.75} \left(e^{-0.75} - 1 \right)$$

$$= \boxed{\frac{5000}{-0.05}\left(e^{0} - e^{0.75}\right)} = 111700.0017$$

Ex: We have an annuity that pays \$1200 per year and earns interest at the annual rate of 8% compounded continuously. How much money are we going to have after 2 years?

$$FV = e^{0.08 \cdot 2} \cdot 1200 \int_0^2 e^{-0.08t}\, dt$$

$$= 1200\, e^{0.16} \frac{1}{-0.08} e^{-0.08t} \Big|_0^2$$

$$= \frac{1200}{-0.08} e^{0.16}\left(e^{-0.16} - 1\right)$$

$$= \boxed{\frac{1200}{-0.08}\left(1 - e^{0.16}\right)} = \boxed{2602.66}$$

Useful life of a machine

Suppose that when it is t years old, a particular machine generate revenue at a rate $R'(t) = 6025 - 8t^2$ dollars per year while the operating and maintanance

at a rate $R(t) = 6025 - 8t^2$ dollars per year, while the operating and maintanance cost accumulate at the rate

$$C'(t) = 4681 + 13t^2 \text{ dollars per year.}$$

a) The useful life of a machine is the number of years before the profit it generates starts to decline.

$$P(t) = R(t) - C(t)$$

To find the useful life of this machine:

$$P'(t) = 0$$

$$R'(t) = C'(t)$$

$$6025 - 8t^2 = 4681 + 13t^2$$

$$\frac{1344}{21} = \frac{21t^2}{21}$$

$$t^2 = 64$$

$$\boxed{t = 8}$$ 8 years.

b) Compute the net profit generated during the useful lifetime.

c) Compare the net profit generated during the useful lifetime.

$$\int_0^8 p'(t)\,dt = \int_0^8 R'(t) - C'(t)\,dt$$

$$= \int_0^8 6025 - 8t^2 - 4681 - 13t^2\,dt$$

$$= \int_0^8 1344 - 21t^2\,dt = 1344t - \frac{21}{3}t^3\Big|_0^8$$

$$= 1344t - 7t^3\Big|_0^8 = 1344\cdot 8 - 7\cdot 8^3 - 0$$

$$= \boxed{7168}$$