12/05

Tuesday, December 5, 2017 2:06 PM

Section 7.3

Formula: $D = f_{xx} \cdot f_{yy} - f_{xy}^2$ is going to be given on the test 4

Classify each crit. pt. of $f(x,y) = x^3 - y^3 + 6xy$ as min/max/saddle pt.

$f_x = 3x^2 + 6y$ $\qquad f_{xx} = 6x$

$f_y = -3y^2 + 6x$ $\qquad f_{yy} = -6y$

$f_{xy} = 6$

$$\begin{cases} 3x^2 + 6y = 0 \longrightarrow & y = -\dfrac{3x^2}{6} = \dfrac{-x^2}{2} \\ -3y^2 + 6x = 0 & \end{cases}$$

$$-3\left(\frac{-x^2}{2}\right)^2 + 6x = 0$$

$$-3 \cdot \frac{x^4}{4} + 6x = 0$$

$$-\frac{3}{4}x^4 + 6x = 0$$

$$-\frac{3}{4}x\left(x^3 - 8\right) = 0$$

$\underline{x = 0}$ $\qquad x^3 = 8$

$x = 2$

$\underline{x = 0}$

$y = \dfrac{-0}{2} = \underline{0}$

$\boxed{(0,0)}$

$\underline{x = 2}$

$y = \dfrac{-2^2}{2} = \underline{-2}$

$x=0$ $x=0$

$x=2$

$y = \frac{}{2} - \underline{}$

$\boxed{(2,-2)}$

$$D(x,y) = 6x\cdot(-6y) - 6^2$$
$$= -36xy - 36$$

$\underline{(0,0):}$ $D(0,0) = -36\cdot0\cdot0 - 36 = \underline{-36 < 0}$

$\boxed{(0,0) \text{ is a saddle pt.}}$

$\underline{(2,-2):}$ $D(2,-2) = -36\cdot2(-2) - 36 = 36\cdot4 - \underline{36 > 0}$

$f_{xx}(2,-2) = 6\cdot2 = \underline{12 > 0}$

$\boxed{(2,-2) \text{ is a rel. minimum}}$

<u>Review</u>

<u>7.3/7</u>

$f = 2x^3 + y^3 + 3x^2 - 3y - 12x - 4$

$f_x = 6x^2 + 6x - 12$

$f_y = 3y^2 - 3$

$f_{xx} = 12x + 6$

$f_{yy} = 6y$

$f_{xy} = 0$

Crit. pts

$$\begin{cases} 6x^2 + 6x - 12 = 0 \to 6(x^2 + x - 2) = 0 \\ 3y^2 - 3 = 0 \end{cases}$$

$6(x+2)(x-1) = 0$

$x = -2, 1$

$3(y^2 - 1) = 0$

$3(y-1)(y+1) = 0$

$y = \pm 1$

Crit. pts: $(-2,-1)$, $(-2,1)$, $(1,-1)$, $(1,1)$

$D = (12x+6)(6y) - 0^2 = 6 \cdot 6(2x+1)(y)$

$= 36(2x+1)y$

$(-2,-1)$: $D(-2,-1) = 36(-4+1) \cdot (-1) > 0$

$f_{yy}(-2,-1) = 6 \cdot (-1) < 0$

$$\boxed{(-2,-1) \text{ is a rel. max}}$$

$\underline{(-2,1)}$: $D(-2,1) = 36(-4+1)\cdot 1 \underline{<0}$

$$\boxed{(-2,1) \text{ is a saddle pt}}$$

$\underline{(1,-1)}$: $D(1,-1) = 36(2+1)\cdot(-1) \underline{<0}$

$$\boxed{(1,-1) \text{ is a saddle pt}}$$

$\underline{(1,1)}$: $D(1,1) = 36(2+1)\cdot 1 \underline{>0}$

$f_{yy} = 6\cdot 1 \underline{>0}$

$$\boxed{(1,1) \text{ is a rel. min}}$$

7.3/29

$$P(x,y) = (40-50x+40y)(x-2)+(20+60x-70y)\cdot(y-2)$$

$$= 40x - 50x^2 + 40xy - 80 \boxed{+10x} + 80y$$

100x

$$+ 20y + 60xy - 70y^2 - 40 - 120x + 140y$$

$$= -50x^2 - 70y^2 - 70x + 80y + 100xy - 120$$

$P_x = -100x - 70 + 100y$ $\qquad$ $P_{xx} = -100$

$P_y = -140y + 80 + 100x$ $\qquad$ $P_{yy} = -140$

$P_{xy} = 100$

Crit. pts: $\begin{cases} 10(-10x - 7 + 10y) = 0 \\ 10(-14y + 8 + 10x) = 0 \end{cases}$

$$\begin{cases} -10x - 7 + 10y = 0 \\ +10x + 8 - 14y = 0 \end{cases}$$

$$0x - 7 + 8 + 10y - 14y = 0 + 0$$

$$1 - 4y = 0$$

$$\boxed{y = \frac{1}{4}}$$

$$-10x - 7 + \frac{10}{4} = 0$$

$$-10x - 7 + \frac{5}{2} = 0$$

$$-10x = 7 - \frac{5}{2} = \frac{14-5}{2} = \frac{9}{2}$$

$$x = \frac{9}{2} \cdot \frac{1}{-10} = \frac{9}{-20} = \underline{-0.45}$$

$$\boxed{(-0.45,\ 0.25)}$$

$$D = -100 \cdot (-140) - 100^2 = 4000 \underline{>0}$$

$$f_{xx} = -100 < 0$$

$$\boxed{(-0.45,\ 0.25) \text{ is a rel. max}}$$

James shirt should be priced as \$2.7, Duncan as \$2.5

7.3 / 16

$$f = (x-4)\ln(xy)$$

$\frac{\partial}{\partial x}(xy)$

$$f_x = \underline{1} \cdot \ln xy + \frac{\cancel{y}}{x \cdot \cancel{y}} \cdot (x-4)$$

$$= \ln xy + \frac{x-4}{x} = \ln xy + 1 - \frac{4}{x}$$

$$= \underline{1 - 4x^{-1} + \ln xy}$$

$$f_y = \frac{\partial}{\partial y}\left((x-4)\ln(xy)\right)$$

$$= (x-4)\frac{\partial}{\partial y}(\ln(xy))$$

$$= (x-4) \cdot \frac{1}{\cancel{x}y} \cdot \cancel{x} = \frac{x-4}{y}$$

$$= xy^{-1} - 4y^{-1}$$

$$= xy^{-1} - 4y^{-1}$$

$$f_{xx} = 0 - 4(-1)x^{-2} + \frac{1}{xy}\cdot y = \boxed{\frac{4}{x^2} + \frac{1}{x}}$$

$$f_{yy} = x\cdot(-1)y^{-2} - 4(-1)y^{-2} = \boxed{\frac{-x}{y^2} + \frac{4}{y^2}}$$

$$f_{xy} = y^{-1} = \boxed{\frac{1}{y}}$$

crit pts:

$$\begin{cases} 1 - \frac{4}{x} + \ln xy = 0 \\ \frac{x-4}{y} = 0 \longrightarrow \boxed{x = 4} \end{cases}$$

$$1 - \frac{4}{4} + \ln(4y) = 0$$

$$\ln(4y) = 0$$

$$4y = 1$$

$$\boxed{y = \frac{1}{4}}$$

$$\boxed{\left(4, \frac{1}{4}\right)}$$

$$D = \left(\frac{4}{x^2} + \frac{1}{x}\right)\left(\frac{-x}{y^2} + \frac{4}{y^2}\right) - \left(\frac{1}{y}\right)^2$$

$$D\left(4, \frac{1}{4}\right) = \left(\frac{\cancel{4}}{\cancel{16}_4} + \frac{1}{4}\right)\left(\frac{-4\cdot 4^2}{1} + \frac{4\cdot 4^2}{1}\right) - 4^2$$

$$= \left(\frac{1}{2}\right) \cdot 0 - 16 = \underline{-16 < 0}$$

$$\boxed{\left(4, \tfrac{1}{4}\right) \text{ is a saddle pt}}$$