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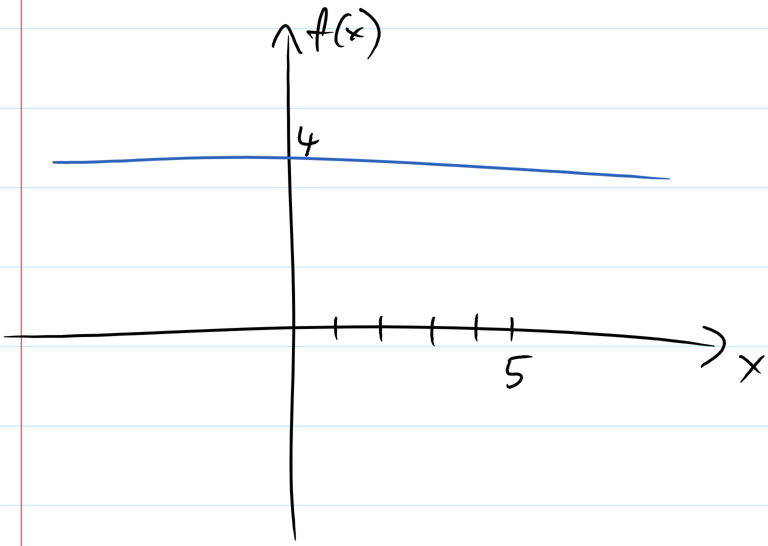
Thursday, August 24, 2017 2:00 PM



$f(x) = 4$

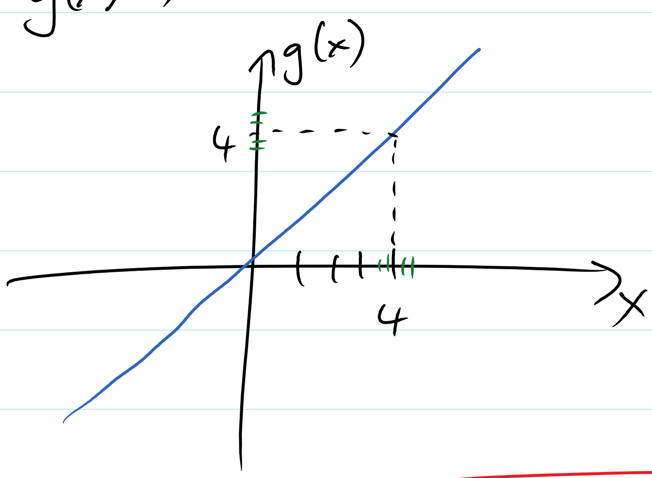
$\lim_{x \to 5} f(x) = 4$

$\lim_{x \to 0} f(x) = 4$

Let $f(x) = k$, then $\lim_{x \to c} f(x) = \boxed{\lim_{x \to c} k = k}$,

where k is a constant.

$g(x) = x$



$\lim_{x \to 4} g(x) = 4$

$\lim_{x \to -3} g(x) = -3$

Observe that $\boxed{\lim_{x \to c} x = c.}$

Limit properties: If $\lim f(x)$ and $\lim g(x)$ exist:

Limit properties: If $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist:

- $\lim_{x \to c} [f(x) \pm g(x)] = [\lim_{x \to c} f(x)] \pm [\lim_{x \to c} g(x)]$
- $\lim_{x \to c} [f(x) \cdot g(x)] = [\lim_{x \to c} f(x)] \cdot [\lim_{x \to c} g(x)]$
- $\lim_{x \to c} [k \cdot f(x)] = k \cdot \lim_{x \to c} f(x)$
- $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, if $\lim_{x \to c} g(x) \neq 0$
- $\lim_{x \to c} [f(x)]^p = [\lim_{x \to c} f(x)]^p$

Ex: $\lim_{x \to -1} (3x^2 - 4x + 8) = \lim_{x \to -1} 3x^2 - \lim_{x \to -1} 4x + \lim_{x \to -1} 8$

$= 3 \cdot \lim_{x \to -1} (x)^2 - 4 \underbrace{\lim_{x \to -1} x}_{= -1} + 8 = 3 \cdot (\underbrace{\lim_{x \to -1} x}_{= -1})^2 - 4 \cdot (-1) + 8$

$= 3 \cdot (-1)^2 + 4 + 8 = \boxed{15}$

$f(x) = 3x^2 - 4x + 8$, $f(-1) = 3 \cdot (-1)^2 - 4 \cdot (-1) + 8 = 15$

Note: To find a limit of a polynomial, evaluate the polynomial.

evaluate the polynomial.

Ex: $\lim\limits_{x\to 1} \dfrac{3x^3-8}{x-2} = 5$

$\dfrac{3\cdot(1)^3-8}{1-2} = \dfrac{3-8}{-1} = 5$

$\lim\limits_{x\to 2} \dfrac{3x^3-8}{x-2}$

$\dfrac{3\cdot 2^3-8}{2-2} = \dfrac{16}{0}$ DNE

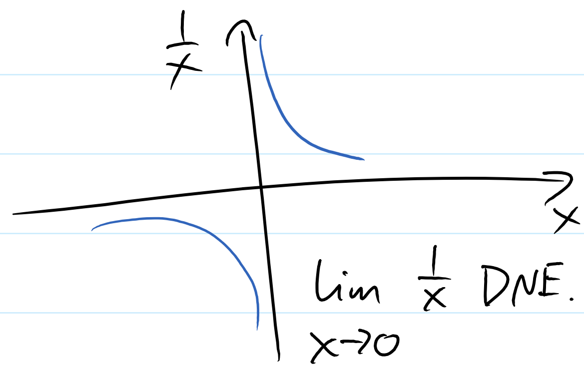
$f(x) = \dfrac{3x^3-8}{x-2}$

$f(2.01) = 1636.18$

$f(1.99) = -1564.18$

$\lim\limits_{x\to 0} \frac{1}{x^2} = +\infty$

b/c the function looks like $\frac{1}{x}$ about 0, the limit DNE.



$\lim\limits_{x\to 0} \frac{1}{x}$ DNE.

Ex: $\lim \dfrac{x^2-1}{x^2-3x+2} = \lim\limits_{x\to 1} \dfrac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x-2)}$

$\dfrac{1-1}{1-3+2} = \dfrac{0}{0}$ not defined

$$\lim_{x\to 1}\frac{x^2-1}{x^2-3x+2} = \lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x-2)}$$

$\frac{}{1-3+2} = 0$

NOT defined.

$$= \lim_{x\to 1}\frac{x+1}{x-2} = \frac{1+1}{1-2} = \boxed{-2}$$

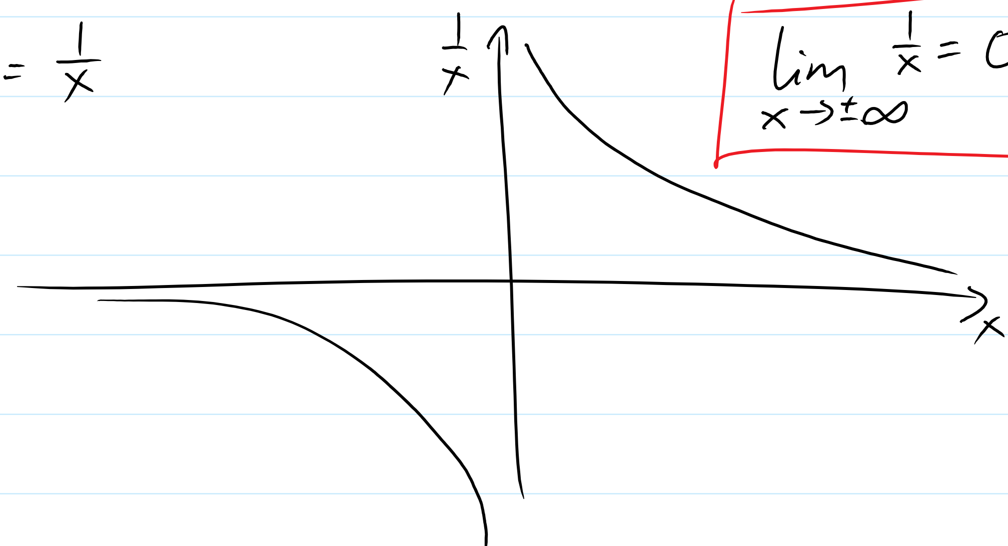
$\frac{0}{0}$ simplify!

Def: If the value of the function $f(x)$ approach the number L as x increases (decreases) without bound, we write

$$\lim_{x\to\infty} f(x) = L.$$

$$\lim_{x\to -\infty} f(x) = L.$$

$f(x) = \frac{1}{x}$



$$\boxed{\lim_{x\to\pm\infty}\frac{1}{x} = 0}$$

Ex:

$$\lim \frac{x^2}{1+x+2x^2}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim \frac{\frac{x^2}{x^2}}{\frac{1}{x^2}+\frac{x}{x^2}+\frac{2x^2}{x^2}}$$

$$\lim_{x\to\infty}\frac{x}{1+x+2x^2}\cdot\frac{x^2}{\frac{1}{x^2}}=\lim_{x\to\infty}\frac{x^2}{\frac{1}{x^2}+\frac{x}{x^2}+\frac{2x^2}{x^2}}$$

$$=\lim_{x\to\infty}\frac{1}{\frac{1}{x^2}+\frac{1}{x}+2}=\frac{\lim_{x\to\infty}1}{\lim_{x\to\infty}\frac{1}{x^2}+\lim_{x\to\infty}\frac{1}{x}+\lim_{x\to\infty}2}$$

$$=\frac{1}{\left(\lim_{x\to\infty}\frac{1}{x}\right)^2+0+2}=\frac{1}{0+0+2}=\boxed{\frac{1}{2}}$$

Ex: $$\lim_{x\to-\infty}\frac{2x^2+3x+1}{3x^2-5x+2}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}=\lim_{x\to-\infty}\frac{2+\cancel{\frac{3}{x}}^{\to 0}+\cancel{\frac{1}{x^2}}^{\to 0}}{3-\cancel{\frac{5}{x}}_{\to 0}+\cancel{\frac{2}{x^2}}_{\to 0}}$$

$$=\frac{2+0+0}{3-0+0}=\boxed{\frac{2}{3}}$$

$$\lim_{x\to 0}\frac{1}{x^2}=\infty$$

Def: We say that $\lim_{x\to c} f(x)$ is a infinite limit if $f(x)$ increases (decreases) without bound as $x\to c$. We write $\lim_{x\to c} f(x)=\infty$.

$$\lim_{x\to c} f(x)=-\infty.$$

Ex: Manufacturing certain comodity, the profit is $P(x) = 4x - \sqrt{x}$ thousand dollars.
What happens the the average profit as the production level goes to 0.

$$AP(x) = \frac{P(x)}{x} = \frac{4x - \sqrt{x}}{x}$$

$$\lim_{x\to 0} \frac{4x-\sqrt{x}}{x} = \lim_{x\to 0}\left(\frac{4x}{x} - \frac{\sqrt{x}}{x}\right) = \lim_{x\to 0} 4 - \frac{1}{\sqrt{x}}$$

$$= \lim_{x\to 0} 4 - \lim_{x\to 0} \frac{1}{\sqrt{x}} = 4 - \lim_{x\to 0} \frac{1}{\sqrt{x}} = 4 - \infty = \boxed{-\infty}$$

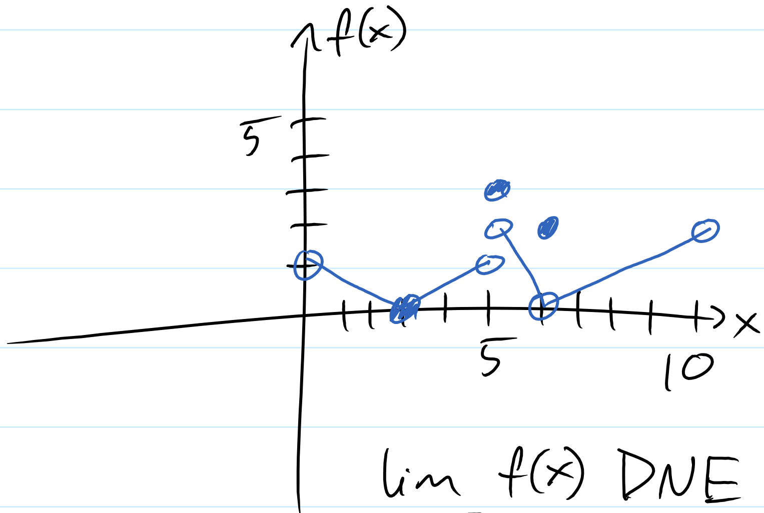
Ex: $$\lim_{x\to\infty} \frac{x^2+x-5}{1-2x-x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$$

$x \to \infty$ so we need to find the degree of denomin and multip. $\frac{1}{x^{\#}}$.

$$= \lim \frac{\cancel{\frac{1}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0} - \cancel{\frac{5}{x^3}}^{\to 0}}{\ldots} = \frac{0+0-0}{0-0-1} = \frac{0}{-1} = \boxed{0}$$

$$= \lim_{x\to\infty} \frac{\cancel{x} + \cancel{x^2} - \cancel{x^3}}{\underbrace{\cancel{\tfrac{1}{x^3}}}_{0} - \underbrace{\cancel{\tfrac{2}{x^2}}}_{0} - 1} = \frac{0+0-0}{0-0-1} = \frac{0}{-1} = \boxed{0}$$

Section 1.6



left hand side lim.

$$\lim_{x\to 5^-} f(x) = 1$$

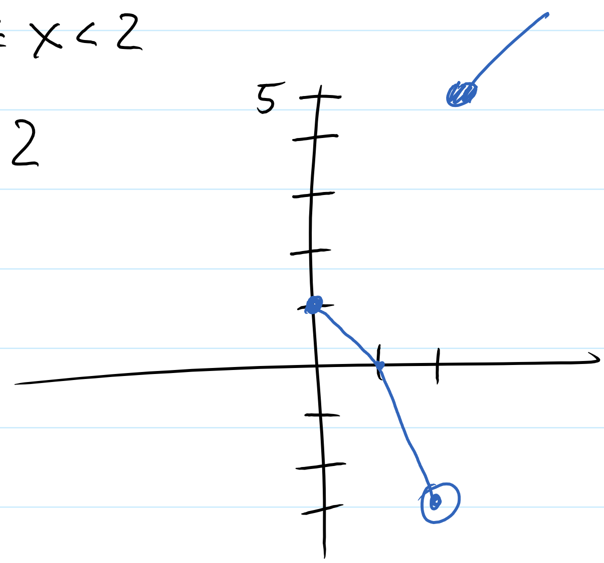
$$\lim_{x\to 5^+} f(x) = 2$$

right hand side lim.

$$\lim_{x\to 5} f(x) \text{ DNE}$$

$$f(5) = 3$$

Ex: $f(x) = \begin{cases} 1-x^2, & 0 \le x < 2 \\ 2x+1, & x \ge 2 \end{cases}$



$$\lim_{x\to 1} f(x) = 1-1^2 = \boxed{0}$$

$$\lim_{x\to 4} f(x) = 2\cdot 4+1 = \boxed{9}$$

$$\lim_{x\to 2} f(x) \ \boxed{\text{DNE}}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 1-x^2 = 1-2^2 = \boxed{-3}$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 2x+1 = 2\cdot 2+1 = \boxed{5}$$

Observation: If $\lim_{x\to c} f(x) = L$, then $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$

Also, if $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$, then $\lim_{x\to c} f(x) = L$.

Ex:

$$f(x) = \begin{cases} x+1, & x<1 \\ -x^2+4x-1, & x\geq 1 \end{cases}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x+1 = 1+1 = \boxed{2}$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} -x^2+4x-1 = -1+4-1 = \boxed{2}$$

$$\lim_{x\to 1} f(x) = \boxed{2}$$

$g(x) = \dfrac{x-2}{x-4}$ Find $\lim_{x\to 4} g(x)$, first $\lim_{x \to 4^-} g(x)$

$$\lim_{x \to 4^-} \frac{x-2}{x-4} = \text{"} \frac{4^- - 2}{4^- - 4} = \frac{2}{0^-} = \boxed{-\infty} \text{"}$$

$3.9 - 4 = -0.1$

$$\lim_{x \to 4^+} \frac{x-2}{x-4} = \text{"} \frac{4-2}{4^+ - 4} = \frac{2}{0^+} = \boxed{+\infty} \text{"}$$

$4.1 - 4 = 0.1$

$$\lim_{x \to 4} \frac{x-2}{x-4} \; \boxed{DNE}$$