8/29

Tuesday, August 29, 2017 2:02 PM

Online Quiz

- due next week
- 60 minutes to complete
- 10 questions
- HW1, HW2
- each question can be answered 3 times.

Section 1.6

$$\lim_{x\to\infty} \frac{2x^3-2x^2+5}{x^2-x+1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{2x-2+\cancel{\frac{5}{x^2}}^{\to 0}}{1-\underbrace{\cancel{\frac{1}{x}}}_{0}+\underbrace{\cancel{\frac{1}{x^2}}}_{0}}$$

$$= \frac{2\cdot\infty-2+0}{1-0+0} = 2\cdot\infty-2 = \infty-2 = \boxed{\infty}$$

$\infty\cdot\infty=\infty$

$(\text{number})\cdot\infty=\infty$

$\infty-(\text{number})=\infty$

$\infty+\infty=\infty$

NO!

$\frac{\infty}{\infty}$, $\infty-\infty$ NOT Defined!

HW 2, #11

$$p(t) = 28 - \frac{6}{t-9}$$

$$c(p) = 0.2\sqrt{p^2 + p - 691}$$

what happens with c as $t \to \infty$

1) $\lim\limits_{t\to\infty} p(t) = \lim\limits_{t\to\infty} 28 - \frac{6}{t-9}$

$= \lim\limits_{t\to\infty} 28 - \lim\limits_{t\to\infty} \frac{6}{t-9} \cdot \frac{\frac{1}{t}}{\frac{1}{t}}$

$= 28 - \lim\limits_{t\to\infty} \frac{\overbrace{\frac{6}{t}}^{\to 0}}{1 - \underbrace{\frac{9}{t}}_{\to 0}} = 28 - \frac{0}{1-0} = 28$

2) $\lim\limits_{t\to\infty} c(p) = \lim\limits_{t\to\infty} c(p(t)) = \ldots$

Def: A function f is continuous at $x=c$ if all three of these conditions are satisfied:

1) $f(c)$ is defined
2) $\lim f(x)$ exists

1) $f(c)$ is defined

2) $\lim_{x\to c} f(x)$ exists

3) $f(c) = \lim_{x\to c} f(x)$.

if the function is not continuous at $x=c$, we say that f is discontinuous at $x=c$.

Ex: $p(x) = 3x^3 - x + 5$, is p continuous at $x=1$?

$$p(1) = 3\cdot 1^3 - 1 + 5 = 3 - 1 + 5 = 7$$

$$\lim_{x\to 1} 3x^3 - x + 5 = 3\cdot 1^3 - 1 + 5 = 3 - 1 + 5 = 7$$

p is continuous at $x=1$.

Note: Polynomials are continuous at any point since the limit is equal the function value.

Ex: Is $f(x) = \dfrac{x+1}{x-2}$ continuous at $x=3$?

$$\left.\begin{array}{l} f(3) = \dfrac{3+1}{3-2} = \dfrac{4}{1} = 4 \\ \lim \dfrac{x+1}{} = \dfrac{3+1}{} = 4 \end{array}\right\} \text{Yes}$$

$$\lim_{x\to 3} \frac{x+1}{x-2} = \frac{3+1}{3-2} = 4$$ Yes

Is it continuous at $x=2$?

No $f(2)$ DNE.

Find points of discontinuity for:

a) $\frac{1}{x}$, $x=0$

b) $\frac{x^2-1}{x+1}$, $x=-1$ function value DNE

c) $h(x)=\begin{cases} x+1, & x<1 \\ 2-x, & x\geq 1 \end{cases}$

Check: $\lim_{x\to 1} h(x) = h(1)$

$h(1) = 2-1 = 1$

$\lim_{x\to 1} h(x)$ DNE

$\lim_{x\to 1^-} h(x) = \lim_{x\to 1^-} x+1 = 2$

$\lim_{x\to 1^+} h(x) = \lim_{x\to 1^+} 2-x = 2-1 = 1$

$\lim_{x \to 1^+} h(x) = \lim_{x \to 1^+} 2 - x = 2 - 1 = 1$)

Ex: let

$$f(x) = \begin{cases} Ax + 5 & , x < 1 \\ x^2 - 3x + 4, & x \geq 1 \end{cases}$$

find the value of A so that $f(x)$ is continuous for all x.

$$\lim_{x \to 1} f(x) = f(1)$$

$$\lim_{x \to 1^-} f(x) = A \cdot 1 + 5 = A + 5$$

$$\lim_{x \to 1^+} f(x) = 1^2 - 3 \cdot 1 + 4 = 1 - 3 + 4 = 2$$

$$A + 5 = 2$$

$$\boxed{A = -3}$$

Def: A function $f(x)$ is continuous on an open interval (a,b) if it is continuous at each point $x = c$ in (a,b).

Also, $f(x)$ is cts on $[a,b]$ if it is continuous on (a,b) and

..., $f(x)$ is cts on $[a,b]$ if f is
continuous on (a,b) and

$$f(a) = \lim_{x \to a^+} f(x) \qquad f(b) = \lim_{x \to b^-} f(x).$$

Is $\frac{1}{x}$ cts on
- $(2, 10)$? ✓
- $[1, 5]$? ✓
- $(0, 1)$? ✓
- $(-1, 2)$? ✗
- $[-3, 0]$? ✗

Is $\frac{x^2-1}{x+1}$ cts on $(-\infty, \infty)$?

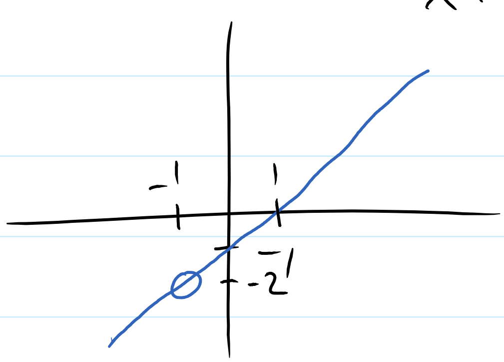
<u>NO!</u>

$$\frac{(-1)^2-1}{-1+1} = \frac{0}{0}$$

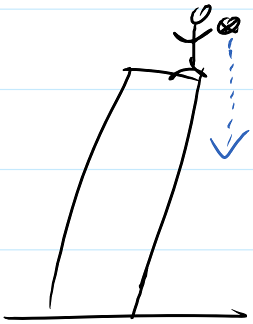
$$\lim_{x \to -1} \frac{x^2-1}{x+1} = \lim_{x \to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}}$$

$$= \lim_{x \to -1} x-1 = -1-1 = -2$$

but $f(x) = \frac{x^2-1}{x+1}$ DNE at $x = -1$.



Section 2.1



If air resistance is neglected, an object dropped from a great height will fall $s(t) = 16t^2$ feet in t seconds.

a) What is the object's velocity after $t=2$ sec.

1) Average velocity $\frac{\text{distance}}{\text{time}}$ on

$h=1$ $[2,3]$: $\frac{s(3)-s(2)}{3-2} = \frac{16\cdot 3^2 - 16\cdot 2^2}{1} = 80$

$h=0.5$ $[2,2.5]$: $\frac{s(2.5)-s(2)}{2.5-2} = 72$

$h=0.01$ $[2,2.01]$: $\frac{s(2.01)-s(2)}{2.01-2} = \frac{16\cdot(2.01)^2 - 16\cdot 2^2}{0.01} = 64.16$

let h be the time diff.

$$\frac{s(2+h)-s(2)}{2+h-2} = \frac{16(2+h)^2 - 16\cdot 2^2}{h}$$

$$\lim_{h\to 0} \frac{16(2+h)^2 - 16\cdot 4}{h} = \lim_{h\to 0} \frac{16\cdot(4+4h+h^2) - 16\cdot 4}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{16\cdot 4} + 16\cdot 4h + 16h^2 - \cancel{16\cdot 4}}{h} = \lim_{h\to 0} \frac{\cancel{h}(16\cdot 4 + 16h)}{\cancel{h}}$$

$$= \lim_{h\to 0} \frac{\quad}{h} = \lim_{h\to 0} \frac{\quad}{\cancel{h}}$$

$$= 16\cdot 4 + 16\cdot 0 = \boxed{64}$$

Def: The derivative of the function $f(x)$ with respect to x is the function $f'(x)$ given by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}.$$

If $f'(c)$ exists, we say that f is differentiable at $x=c$.

Ex: $f(x)=3x^2$. Find $f'(x)$, $f'(1)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2 - 3\cdot x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6x\cancel{h} + 3h^{\cancel{2}} - \cancel{3x^2}}{\cancel{h}} = \lim_{h\to 0} 6x+3h = \boxed{6x}$$

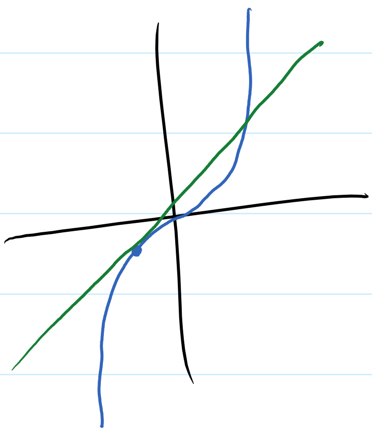
$$f'(1) = 6\cdot 1 = \boxed{6}$$

Derivative at $x=c$ is the instantaneous rate of change at $x=c$.

rate of change at $x=c$.

$y=f(x)$, $f'(c)$ is the slope of the tangent line at $(c, f(c))$.

Ex: Find the tangent line to $y=x^3$ at $x=-1$.



$$y(-1)=(-1)^3=-1$$

$$(-1,-1)$$

$$y'(x)=\lim_{h\to 0}\frac{y(x+h)-y(x)}{h}=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2\cancel{h}+3xh^{\cancel{2}}+h^{\cancel{3}2}-\cancel{x^3}}{\cancel{h}}$$

$$=\lim_{h\to 0} 3x^2+3xh+h^2=3x^2$$

$y'(-1)=3\cdot(-1)^2=3$ ← slope

$$y-y_0=m(x-x_0) \quad \Big| \quad y=mx+b$$

$$y-(-1)=3(x-(-1))$$

$$y+1=3(x+1)=3x+3$$

$$\boxed{y=3x+2}$$

$$f'(x)=\lim_{c\to x}\frac{f(c)-f(x)}{c-x}$$

$$\boxed{f'(x) = \lim_{c \to x} \frac{f(c) - f(x)}{c - x}}$$

$$f(x) = x^n$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$