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Thursday, August 31, 2017 1:58 PM

Ex:

$$\lim_{x\to 9^+} \frac{x-9}{3-\sqrt{x}} \cdot \frac{3+\sqrt{x}}{3+\sqrt{x}} = \lim_{x\to 9^+} \frac{(x-9)(3+\sqrt{x})}{-(9+x)}$$

$\frac{0}{0}$

$$= \lim_{x\to 9^+} \frac{\cancel{(x-9)}(3+\sqrt{x})}{-\cancel{(x-9)}} = \frac{3+\sqrt{9}}{-1} = \frac{6}{-1} = \boxed{-6}$$

Ex: $f(x) = \begin{cases} x, & \text{if } x<1 \\ x^2 - x, & \text{if } x \geq 1 \end{cases}$

- is f continuous at $(0,1)$
  $0 < x < 1$

Yes b/c $f(x) = x$ at $(0,1)$

- if f cts at $[0,1]$
  f is cts at $x=0$ b/c $f(x) = x$ there.
- $\underbrace{\text{is } f \text{ cts at } x=1?}_{f(1) = \lim_{x\to 1^-} f(x)}$

$$\left.\begin{array}{l} f(1) = 1^2 - 1 = 0 \\ \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1 \end{array}\right\} \text{The function is not cts on } [0,1].$$

Section 2.1 cont

Ex: The profit function is

$$P(x) = -400x^2 + 6800x - 12000,$$

where x ... thousands of units.

At what rate should we expect profit to be changing with respect to the level of production x, when 9000 units are produced? Is the profit increasing or decreasing at the level of production?

Find $P'(9)$.

$$P'(x) = \lim_{h \to 0} \frac{P(x+h) - P(x)}{h}$$

$$= \lim_{h \to 0} \frac{-400(x+h)^2 + 6800(x+h) - 12000 - (-400x^2 + 6800x - 12000)}{h}$$

$$= \lim_{h \to 0} \frac{-400(x+h)^2 + \cancel{6800x} + 6800h \cancel{-12000} + 400x^2 \cancel{-6800x} \cancel{+12000}}{h}$$

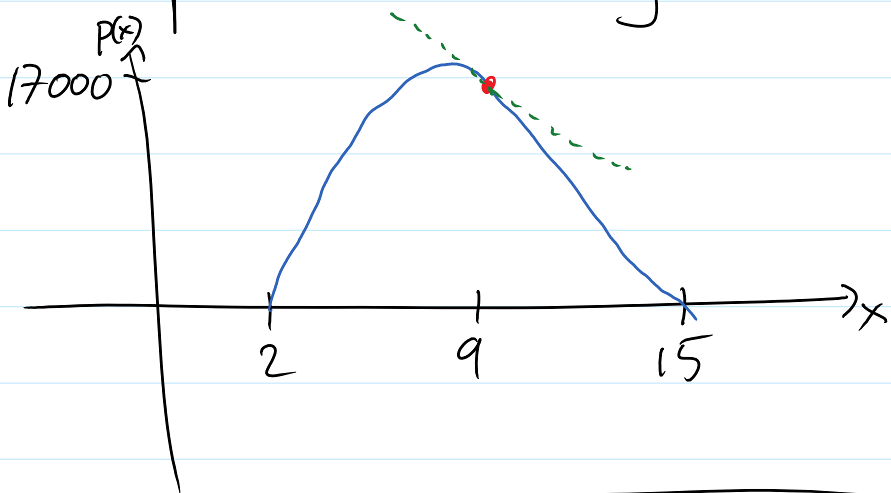
$$= \lim_{h \to 0} \frac{-400 \cdot 2x\cancel{h} - 400h^{\cancel{2}} + 6800\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} -800x - 400h + 6800 = -800x + 6800$$

$$P'(9) = -800 \cdot 9 + 6800 = -7200 + 6800 = \boxed{-400}$$

The profit is changing by -400 \$/1000 units.

The profit is decreasing.



Thm: If the function f is differentiable at $x = c$, then

f is increasing (decreasing) at $x = c$,

if $f'(c) > 0$ $(f'(c) < 0)$

Notation:

If f is a function, the following are equivalent

$$f'(x) = \frac{df}{dx}$$

$$y = f(x), \text{ then } y' = f'(x) = \frac{dy}{dx}$$

$$f'(c) = y'(c) = f'(x)\Big| \quad = \frac{dy}{dx}\Big|$$

$$f'(c) = y'(c) = f'(x)\Big|_{x=c} = \frac{dy}{dx}\Big|_{x=c}$$

<u>Ex:</u> $f(x)=\sqrt{x}$, find the equation of the tangent line at $x=4$.

First find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{2\cdot\sqrt{4}} = \frac{1}{4}$$ ← slope of the tan. line

point ↙

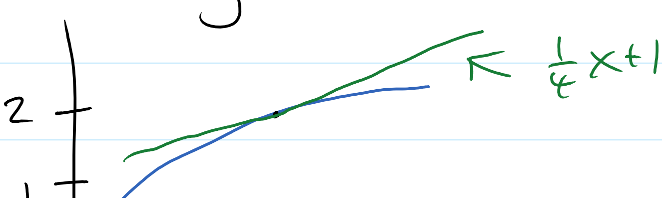
$x=4$, $f(4)=\sqrt{4}=2$ $\quad (4,2)$

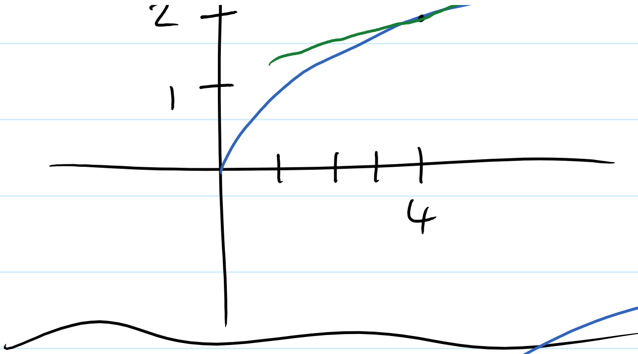
$$y-y_0 = m(x-x_0)$$

$$y-2 = \frac{1}{4}(x-4)$$

$$y = \frac{1}{4}x - 1 + 2$$

$$\boxed{y = \frac{1}{4}x + 1}$$





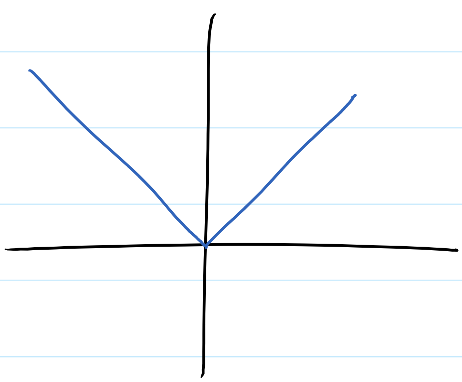
domain: $[0, \infty)$

Is $f(x) = \sqrt{x}$ differentiabe on $(0, \infty)$? Yes

$f'(x) = \frac{1}{2\sqrt{x}}$ ← domain: $(0, \infty)$

$f'(0)$ DNE, i.e., $\sqrt{x}$ is not differentiable at $x=0$.

The function $|x|$ is continuous on $(-\infty, \infty)$ but it is not differentiable at $x=0$.



Thm: If the function $f(x)$ is differentiable at $x=c$, then it is also continuous at $x=c$.

Section 2.2: Differentiation rules.

① Derivative of a constant function.

$f(x) = a$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{a-a}{h} = \lim_{h\to 0} \frac{0}{h}$$

$$= \lim_{h\to 0} 0 = \boxed{0}$$

$$\frac{d}{dx}(c) = (c)' = 0.$$

the derivative of ...

② Derivative of x^n.

$f(x) = x^n$, n is positive integer.

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^n - (x)^n}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + \binom{n}{1}x^{n-1}\cancel{h} + \binom{n}{2}x^{n-2}h^{\cancel{2}} + \dots + h^{\cancel{n}\,n-1} - \cancel{x^n}}{\cancel{h}}$$

$$= \lim_{h\to 0} \binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2}h + \binom{n}{3}x^{n-3}h^2 + \dots + \binom{n}{n}h^{n-1}$$

$$= \binom{n}{1}x^{n-1} + 0 + 0 + \dots + 0$$

$$= \boxed{n x^{n-1}}$$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$