9/05

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Section 2.2

Find the derivative of $f(x) = x^4$, $n=4$

$$f'(x) = \frac{df}{dx} = 4 \cdot x^{4-1} = \boxed{4x^3}$$

$$g(x) = x^{5.5}, \quad g'(x) = ?$$

$$g'(x) = \boxed{5.5x^{4.5}}$$

$$h(x) = \sqrt{x} = x^{\frac{1}{2}}, \quad h'(x) = ?$$

$$h'(x) = \boxed{\frac{1}{2\sqrt{x}}}$$

$$h'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{1/2}} = \frac{1}{2 \cdot \sqrt{x}}$$

The power rule:

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}, \quad \text{if } n \neq 0.$$

The constant multiple rule:

$$\frac{d}{dx}(cf(x)) = c \cdot \frac{d}{dx}(f(x)), \quad \text{where } c \text{ is a constant}$$

a constant

$$(c f(x))' = c \cdot f'(x)$$

The sum/difference rule:

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

Ex: Differenciate: $f(x) = 3x^4 - 5x$

$$(3x^4 - 5x)' = (3x^4)' - (5x)' = 3(x^4)' - 5(x)'$$
$$= 3 \cdot 4x^3 - 5 \cdot 1 \cdot x^0 = \boxed{12x^3 - 5}$$

• $g(x) = 2x^5 - 3x^{-7}$

$$g'(x) = 2 \cdot (x^5)' - 3(x^{-7})' = 2 \cdot 5x^4 - 3 \cdot (-7)x^{-8}$$
$$= \boxed{10x^4 + 21x^{-8}}$$

• $h(x) = \dfrac{x - x^2 + 5}{x} = \dfrac{x}{x} - \dfrac{x^2}{x} + \dfrac{5}{x} = 1 - x + 5x^{-1}$

$$h'(x) = (1)' - (x)' + 5(x^{-1})' = 0 - 1 + 5 \cdot (-1)x^{-2}$$
$$= \boxed{-1 - 5x^{-2}}$$

$h'(x) = (1)' - (x)' + 5(x^{-1})' = 0 - 1 + 5(-1)x$

$= \boxed{-1 - 5x^{-2}}$

$= \boxed{-1 - \frac{5}{x^2}}$

Def: The relative rate of change of $Q(x)$ is

$$\frac{Q'(x)}{Q(x)}.$$

The percentage rate of change of Q is

$$\frac{Q'(x)}{Q(x)} \cdot 100$$

Ex:

The GDP of a country was $N(t) = t^2 + 5t + 106$ billion dollars t years after 2000.

a) At what rate was the GDP changing with respect to time in 2010?

$$N'(t) = 2t + 5 \cdot 1 + 0 = 2t + 5$$

$$N'(10) = 2 \cdot 10 + 5 = \boxed{25} \text{ bill. \$/year}$$

b) The percentage rate of change in 2010.

$$100 \cdot \frac{N'(10)}{N(10)} = 100 \cdot \frac{25}{10^2+5\cdot 10+106} = \boxed{9.76\%}$$

In phisics:

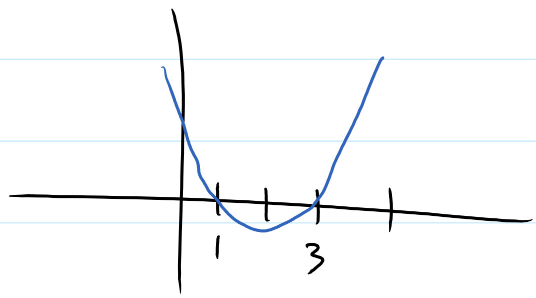
s(t)... displacement
v(t)... velocity
a(t)... acceleration

$\boxed{s'(t) = v(t)}$ | $\boxed{v'(t) = a(t)}$

Ex: The position of an object is given by
$s(t) = t^3 - 6t^2 + 9t + 5$

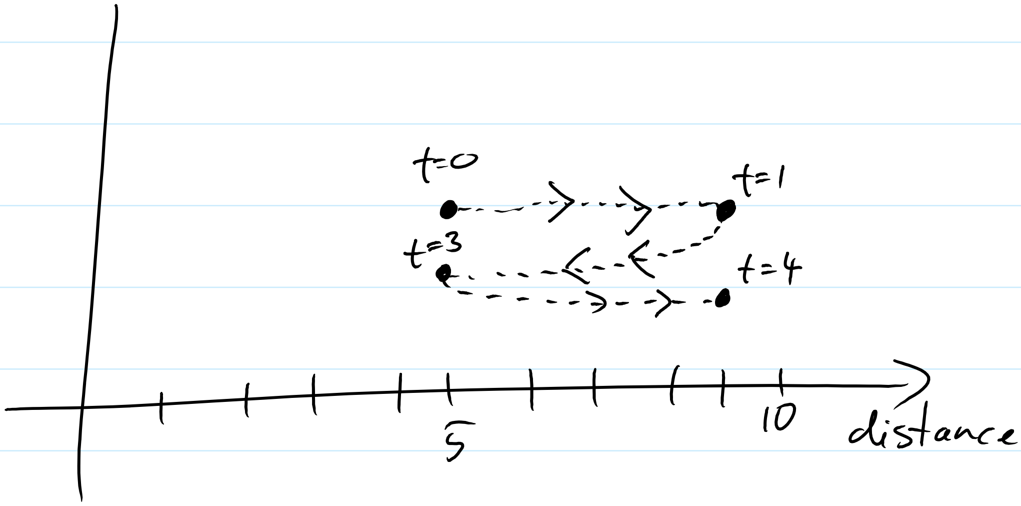
a) Find the velocity of the object moving along a line between times $t=0$ and $t=4$.

$$v(t) = s'(t) = 3t^2 - 6\cdot 2t + 9 + 0$$
$$= 3t^2 - 12t + 9 = 3(t^2 - 4t + 3)$$
$$= 3(t-3)(t-1)$$



b) Find the total distance traveled by the

b) Find the total distance traveled by the object between times $t=0$ and $t=4$



$s(t) = t^3 - 6t^2 + 9t + 5$ | $s(0) = 5$ | $s(1) = 1 - 6 + 9 + 5 = 9$

$s(3) = 27 - 54 + 27 + 5 = 5$

$s(4) = 64 - 6 \cdot 16 + 36 + 5 = 9$

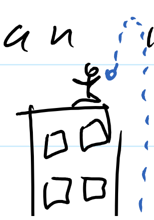
The total distance is 12 units.

a) Find the object's acceleration.

$v(t) = 3t^2 - 12t + 9$ | $a(t) = v'(t)$

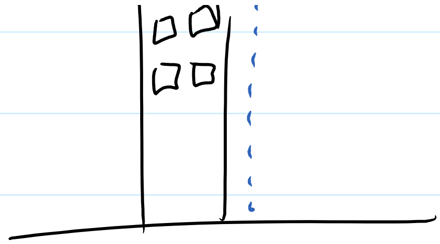
$= 3 \cdot 2t^1 - 12 = \boxed{6t - 12}$

Ex: Suppose a person standing at the top of a building 112 feet high throws a ball vertically upward with an initial velocity of 96 ft/sec.



height acceler init. velocity

$$\boxed{H(t) = -\tfrac{1}{2} g t^2 + V_0 t + H_0}$$



$g = 32 \mid V_0 = 96, \; H_0 = 112$

$$H(t) = -\tfrac{1}{2}\cdot 32 t^2 + 96t + 112 = -16t^2 + 96t + 112$$

a) Find when does the ball hit the ground and the impact velocity.

$$-16t^2 + 96t + 112 = 0$$
$$-16(t^2 - 6t - 7) = 0$$
$$-16(t-7)(t+1) = 0$$
$$t = \boxed{7}, \cancel{-1}$$

$$v(t) = -16\cdot 2t + 96$$
$$= -32t + 96$$

$$v(7) = -224 + 96 = \boxed{-128}$$

velocity is -128 ft/s

b) When is the velocity 0?

$$v(t) = 0$$
$$-32t + 96 = 0$$
$$-32t = -96$$
$$t = 96/32 = 48/16 = 24/8 = \boxed{3}$$

c) How far does the ball travel during its fight?

$$s(3) = -16\cdot 3^2 + 96\cdot 3 + 112 = \boxed{256 \text{ ft}}$$

Section 2.3

$f(t) = (t+3)(t-2)$, find $f'(t)$

$= t^2 + t - 6$ $\qquad f'(t) = 2t+1$

$g(x) = (x+3)(x-2)^2$, find $g'(x) = 3x^2 - 2x - 8$

$= (x+3)(x^2 - 4x + 4)$

$= x^3 - x^2 - 8x + 12$

The product rule:

$$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + g'(x)\cdot f(x)$$

$$(\underbrace{(t+3)}_{f}\underbrace{(t-2)}_{g})' = (t+3)'(t-2) + (t-2)'\cdot(t+3)$$

$$= 1\cdot(t-2) + 1\cdot(t+3)$$

$$= t - 2 + t + 3 = \underline{2t+1}$$

Proof:

$$\frac{d}{dx}\left(f(x)g(x)\right) = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h) - f(x)]\, g(x+h)}{h} + \frac{f(x)[g(x+h) - g(x)]}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \cdot \lim_{h\to 0} g(x+h) + f(x) \cdot \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) \cdot g(x+0) + f(x) \cdot g'(x)$$

$$= f'(x)g(x) + g'(x)f(x)$$

The quotient rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

$$\left(\frac{hi}{lo}\right)' = \frac{lo \cdot dhi - hi \cdot dlo}{(lo)^2}$$

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differentiate:

$$P(x) = \overbrace{(x-1)}^{f}\overbrace{(2x^2 - x - 1)}^{g}$$

$$P'(x) = (x-1)'(2x^2 - x - 1) + (2x^2 - x - 1)'(x-1)$$

$$P'(x) = (x-1)'(2x^2-x-1) + (2x^2-x-1)'(x-1)$$
$$= 1\cdot(2x^2-x-1) + (2\cdot 2x-1)\cdot(x-1)$$
$$= 2x^2-x-1+4x^2-4x-x+1$$
$$= \boxed{6x^2-6x}$$

Find all the points where the tangent line is horizontal.

$$P'(x)=0$$
$$6x^2-6x=0$$
$$6x(x-1)=0$$
$$\boxed{x=0,1}$$

Diff:

$$Q(x) = \frac{x^2-5x+7}{2x} = \frac{1}{2}x - \frac{5}{2} + \frac{7}{2}x^{-1}$$

$$Q'(x) = \frac{1}{2} - 0 + \frac{7}{2}(-1)x^{-2} = \boxed{\frac{1}{2} - \frac{7}{2x^2}}$$

$$Q'(x) = \frac{2x\cdot(x^2-5x+7)' - (x^2-5x+7)(2x)'}{(2x)^2}$$

$$= \frac{\cancel{2}x(2x-5) - (x^2-5x+7)\cdot\cancel{2}}{{}^{2}\cancel{4}x^2}$$

$$= \frac{2x^2\cancel{-5x} - x^2 + \cancel{5x} - 7}{2x^2} = \frac{x^2-7}{2x^2} = \frac{1}{2} - \frac{7}{2x^2}$$