Section 2.4

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dy}(y(u(x))) = y'_{(u)} \cdot u'_{(x)}$$

Ex: $y = \sqrt{x^2+3x+2}$

$y = \sqrt{u}$ $\qquad u = x^2+3x+2$

$$y'(x) = \frac{d}{du}(\sqrt{u}) \cdot \frac{d}{dx}(u(x))$$
$$= (u^{1/2})' \cdot (x^2+3x+2)'$$
$$= \frac{1}{2}u^{-\frac{1}{2}} \cdot (2x+3+0) = \frac{1}{2} \cdot \frac{1}{u^{1/2}} \cdot (2x+3)$$
$$= \frac{2x+3}{2 \cdot \sqrt{u}} = \boxed{\frac{2x+3}{2 \cdot \sqrt{x^2+3x+2}}}$$

Ex: $f(x) = (2x^4-x)^3$, Find $f'(x)$.

$= (u)^3$, $u = 2x^4-x$

$$f'(x) = \frac{d}{du}(u^3) \cdot \frac{d}{dx}(2x^4 - x) = (u^3)' \cdot (2x^4 - x)'$$
$$= 3u^2 \cdot (2 \cdot 4x^3 - 1)$$
$$= \boxed{3(2x^4 - x)^2 (8x^3 - 1)}$$

$$f(x) = \sqrt{6 + \frac{1}{3x}} = \left(6 + (3x)^{-1}\right)^{1/2}$$

$u^{1/2} \to \frac{1}{2} u^{-1/2} = \frac{1}{2\sqrt{u}}$

$u = 6 + \frac{1}{3}x^{-1}$

$$= \left(6 + \frac{1}{3}x^{-1}\right)^{1/2}$$

$$f'(x) = \frac{1}{2\sqrt{6 + \frac{1}{3}x^{-1}}} \cdot \left(6 + \frac{1}{3}x^{-1}\right)'$$

$$= \frac{1}{2\sqrt{6 + \frac{1}{3}x^{-1}}} \cdot \left(0 + \frac{1}{3} \cdot (-1)\, x^{-2}\right)$$

$$= \boxed{\frac{-\frac{1}{3}x^{-2}}{2\sqrt{6 + \frac{1}{3}x^{-1}}}} = \boxed{\frac{-1}{2 \cdot 3x^2 \sqrt{6 + \frac{1}{2}x^{-1}}}}$$

$$f(x) = \underbrace{(3x+1)^4}_{f}\underbrace{(2x-1)^5}_{g}$$

$(f \cdot g)' = gf' + fg'$

$$= (2x-1)^5 \left[(3x+1)^4\right]' + (3x+1)^4 \cdot \left[(2x-1)^5\right]'$$

use chain rule:

$[u^4]' = 4u^3$ $\qquad$ $[u^5]' = 5u^4$

use chain rule:

$[u^4]' = 4u^3$, $u = 3x+1$ $\qquad$ $[u^5]' = 5u^4$, $u = 2x-1$

$$= (2x-1)^5 \underbrace{4\cdot(3x+1)^3}_{4u^3}\cdot(3x+1)' + (3x+1)^4 \underbrace{5\cdot(2x-1)^4}_{5u^4}\cdot(2x-1)'$$

$$= (2x-1)^5 4(3x+1)^3 3 + (3x+1)^4 5(2x-1)^4 \cdot 2$$

$$= 2(2x-1)^4(3x+1)^3[6(2x-1)+5(3x+1)]$$

$$= 2(2x-1)^4(3x+1)^3[12x-6+15x+5]$$

$$= \boxed{2(2x-1)^4(3x+1)^3(27x-1)}$$

Ex:

Diff: $f(x) = \dfrac{3x-2}{(x-1)^2}$ $\qquad$ $[u^2]' = 2u\cdot u' = 2\cdot(x-1)\cdot(x-1)'$

$$f'(x) = \frac{(x-1)^2\cdot(3x-2)' - (3x-2)[(x-1)^2]'}{[(x-1)^2]^2}$$

$$= \frac{(x-1)^2 3 - (3x-2)2(x-1)\cdot(x-1)'}{(x-1)^4}$$

$$= \frac{3(x-1)^2 - 2(3x-2)(x-1)}{(x-1)^4} = \frac{\cancel{(x-1)}[3(x-1)-2(3x-2)]}{(x-1)^{4}3}$$

$$= \frac{3x-3-6x+4}{(x-1)^3} = \boxed{\frac{-3x+1}{(x-1)^3}}$$