Section 2.1

a) Find the average ~~value~~ rate of change of $s(t) = \frac{t-1}{t+1}$ when t changes from $t = -\frac{1}{2}$ to $t = 0$

average ~~value~~ rate of change formula: $\frac{s(t_2) - s(t_1)}{t_2 - t_1}$

$$\frac{s(0) - s(-\frac{1}{2})}{0 - (-\frac{1}{2})} = \frac{\frac{0-1}{0+1} - \frac{-\frac{1}{2}-1}{-\frac{1}{2}+1}}{\frac{1}{2}} = \frac{-1 - \frac{-3/\cancel{2}}{1/\cancel{2}}}{1/2} = \frac{-1-(-3)}{\frac{1}{2}}$$

$$= \frac{2}{\frac{1}{2}} = 2 \cdot \frac{2}{1} = \boxed{4}$$

b) Find the eq. of the tang. line to $y = \frac{-2}{x}$ at $x = 1$.

(use the def. of derivative)

$$y' = \lim_{h \to 0} \frac{y(x+h) - y(x)}{h} = \lim_{h \to 0} \frac{\frac{-2}{x+h} - \left(\frac{-2}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-2}{x+h} \cdot \frac{x}{x} + \frac{2}{x} \cdot \frac{x+h}{x+h}}{h} = \lim_{h \to 0} \frac{\frac{-2x + 2(x+h)}{(x+h)x}}{h}$$

$$= \lim \frac{\cancel{-2x} + \cancel{2x} + 2h}{h(x+h)x} = \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}(x+h)x} = \frac{2}{(x+0)x} = \frac{2}{x^2}$$

$$= \lim_{h\to 0} \frac{-\cancel{2x}+\cancel{2x}+2h}{h(x+h)x} = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(x+h)x} = \frac{2}{(x+0)x} = \frac{2}{x^2}$$

$$y'(1) = \frac{2}{1^2} = 2 \quad \text{slope} \qquad y(1) = \frac{-2}{1} = -2$$

$$y-(-2) = 2(x-1)$$
$$y+2 = 2x-2 \quad \rightarrow \quad \boxed{y = 2x-4}$$

$\boxed{2.2}$ Find the tangent line to $f(x) = x - \sqrt{x} + \frac{1}{x^2}$, $x=1$.

$$f(x) = x - x^{1/2} + x^{-2} \qquad f(1) = 1 - \sqrt{1} + \frac{1}{1} = \boxed{1}$$

slope:

$$f'(x) = 1 - \frac{1}{2}x^{-1/2} + (-2)x^{-3} = 1 - \frac{1}{2}\cdot\frac{1}{\sqrt{x}} - \frac{2}{x^3}$$
$$f'(1) = 1 - \frac{1}{2}\cdot\frac{1}{\sqrt{1}} - \frac{2}{1^3} = 1 - \frac{1}{2} - 2 = -1.5 = -\frac{3}{2}$$

$$y-1 = -\frac{3}{2}(x-1)$$
$$y-1 = -\frac{3}{2}x + \frac{3}{2}$$
$$\boxed{y = -\frac{3}{2}x + 5/2}$$

b) Differenciate:

$$y = -\frac{7}{x^{1.2}} + \frac{5}{x^{-2.1}}$$

Think of $x^{-2} = \frac{1}{x^2}$

$\frac{1}{x^3} = x^{-3}$

$$= -7\cdot x^{-1.2} + 5x^{-(-2.1)}$$
$$= -7x^{-1.2} + 5x^{2.1}$$

Power rule:

$$\frac{d}{dx}(x^n) = n\cdot x^{n-1}$$
$$(x^n)' = n\cdot x^{n-1}$$

$$y' = -7\cdot(-1.2)x^{-2.2} + 5\cdot(2.1)x^{1.1}$$

$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$
$(x^n)' = n \cdot x^{n-1}$

$$y' = -7 \cdot (-1.2)\, x^{-2.2} + 5 \cdot (2.1)\, x^{1.1}$$
$$= \boxed{8.4 \cdot x^{-2.2} + 10.5\, x^{1.1}}$$

$$\bullet\ y = \frac{x^5 - 4x^2}{x^3} = \cancel{x^{5} - 4x^{2-3}} \quad \frac{x^5}{x^3} - \frac{4x^2}{x^3} = x^{5-3} - 4x^{2-3}$$
$$= x^2 - 4x^{-1}$$
$$y' = 2 \cdot x^1 - 4 \cdot (-1)\, x^{-2} = \boxed{2x + 4x^{-2}}$$

$\boxed{2.3}$

a) Find the tangent line at the indicated point:

1) $y = \underbrace{(x^2+3)}_{f}\underbrace{(5-2x^3)}_{g}, \quad x_0 = 1$

$(f \cdot g)' = g f' + f g'$

$$y' = (5-2x^3) \cdot (x^2+3)' + (x^2+3)(5-2x^3)'$$
$$= (5-2x^3) \cdot 2x + (x^2+3)(-6x^2)$$

$x = 1$ $\qquad \boxed{(1,12)}$

$y(1) = (1+3)(5-2) = 12$

$$y'(1) = (5-2\cdot1^3)\cdot 2\cdot 1 + (1+3)(-6\cdot 1^2) = 3\cdot 2 + 4\cdot(-6)$$
$$= 6 - 24 = -18$$

$$y - 12 = -18(x-1)$$
$$y = -18x + 18 + 12$$
$$\boxed{y = -18x + 30}$$

$$\boxed{y = -18x + 30}$$

$$2)\quad y = x + \frac{3}{2-4x}, \quad x_0 = 0$$

$$= x + 3(2-4x)^{-1}$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$y' = 1 + \frac{(2-4x)(3)' - 3(2-4x)'}{(2-4x)^2} = 1 + \frac{(2-4x)0 - 3\cdot(-4)}{(2-4x)^2}$$

$$= 1 + \frac{12}{(2-4x)^2}$$

$$y'(0) = 1 + \frac{12}{(2-0)^2} = 1 + \frac{12}{4} = 1 + 3 = 4$$ slope

$$\left(0, \tfrac{3}{2}\right)$$

$$y(0) = 0 + \frac{3}{2-4\cdot 0} = \frac{3}{2}$$

$$y - \tfrac{3}{2} = 4(x-0)$$

$$\boxed{y = 4x + 3/2}$$

Differenciate: $f(t) = \left(t + \frac{1}{t}\right)^2 = (t + t^{-1})^2 = \cancel{t^2 + t^{-2}}$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= t^2 + 2\cdot t \cdot t^{-1} + (t^{-1})^2$$

$$= t^2 + 2\cdot \cancel{t}\cdot \frac{1}{\cancel{t}} + t^{-2} = t^2 + 2 + t^{-2}$$

$$f'(t) = 2t + 0 + (-2)t^{-3} = \boxed{2t - 2t^{-3}}$$

Using the chain rule: $\frac{d}{dt}\left(\left(\underbrace{t^2 + t^{-1}}\right)^2\right)$

Using the chain rule: $\frac{d}{dt}\left(\left(t^2+t^{-1}\right)^2 \right)$

$u = t^2 + t^{-1}$

$y = u^2$

$$f'(t) = \underbrace{2\left(t^2+t^{-1}\right)^1}_{2u} \cdot \left(t^2+t^{-1}\right)' = 2\left(t^2+t^{-1}\right)\cdot\left(2t+(-1)t^{-2}\right)$$

$$= \boxed{2\left(t^2+t^{-1}\right)\left(2t-t^{-2}\right)}$$

Section 2.4

Differenciate: $f(x) = 3\underbrace{(4x+1)^4}_{f} \cdot \underbrace{(9x-4)^3}_{g}$

$(f\cdot g)' = gf' + fg'$

$$f'(x) = 3\left[(9x-4)^3 \cdot \left[(4x+1)^4\right]' + (4x+1)^4\left[(9x-4)^3\right]' \right]$$

$$\frac{d}{dx}\left((f(x))^n \right) = n(f(x))^{n-1} \cdot f'(x)$$

$$= 3(9x-4)^3 \cdot 4(4x+1)^3(4x+1)' + 3(4x+1)^4 3(9x-4)^2 \cdot (9x-4)'$$

$$= 3(9x-4)^3 \cdot 4(4x+1)^3 \cdot 4 + 3(4x+1)^4 \cdot 3 \cdot (9x-4)^2 \cdot 9$$

$$= 48(9x-4)^3(4x+1)^3 + 81(9x-4)^2(4x+1)^4$$

$$= 3(9x-4)^2(4x+1)^3\left[16(9x-4) + 27(4x+1)\right]$$

$$= 3(9x-4)^2(4x+1)^3\left[144x - 64 + 108x + 27\right]$$

$$= \boxed{3(9x-4)^2(4x+1)^3\left[252x - 37\right]}$$

Limits

$$\lim \frac{x^3+5x}{(x-2)(2x+3)}$$

$$\frac{8+10}{0(4+3)} = \frac{18}{0}$$

$$\lim_{x \to 2} \frac{x^3+5x}{(x-2)(2x+3)}$$ $$\frac{8+10}{0(4+3)} = \frac{18}{0}$$

Have to check one sided limits.

$$\lim_{x \to 2^+} \frac{x^3+5x}{(x-2)(2x+3)}$$ $$\frac{8+10}{(2.1-2)(4+3)} = \frac{18}{(0.1)\cdot 7} = \boxed{+\infty}$$

2.1

positive

$$\lim_{x \to 2^-} \frac{x^3+5x}{(x-2)(2x+3)}$$ $$\frac{18}{(1.9-2)7} = \frac{18}{(-0.1)7} = \boxed{-\infty}$$

1.9

negative

different, so the $\lim_{x \to 2}$ $\boxed{\text{DNE}}$