Section 2.5

Def: Marginal Cost: If $C(x)$ is the total cost function of producing x units of a commodity, then the marginal cost of producing x_0 units is the derivative, $C'(x_0)$.

For large x_0, the marginal cost $C'(x_0)$ can be used to estimate the additional cost $C(x_0+1) - C(x_0)$ incurred when the level of production is increased from x_0 to x_0+1.

· Similarly, we have the marginal revenue, $R'(x)$ and the marginal profit, $P'(x)$.

EXAMPLE 2.5.1 Studying Marginal Cost and Marginal Revenue

A manufacturer estimates that when x units of a particular commodity are produced, the total cost will be $C(x) = \frac{1}{8}x^2 + 3x + 98$ dollars, and furthermore, that all x units will be sold when the price is $p(x) = \frac{1}{3}(75 - x)$ dollars per unit.

a. Find the marginal cost and the marginal revenue.

b. Use marginal cost to estimate the cost of producing the 37th unit. What is the actual cost of producing the 37th unit?

c. Use marginal revenue to estimate the revenue derived from the sale of the 37th unit. What is the actual revenue derived from the sale of the 37th unit?

Ex: $C(x) = \frac{1}{8}x^2 + 3x + 98$, $p(x) = \frac{1}{3}(75 - x)$

Ex: $C(x)=\frac{1}{8}x^2+3x+98$, $p(x)=\frac{1}{3}(75-x)$

a) $\boxed{C'(x)=\frac{1}{4}x+3}$

$R(x)=p(x)\cdot x=\frac{1}{3}(75-x)x$

$=\frac{1}{3}(75x-x^2)$

$R'(x)=\boxed{\frac{1}{3}(75-2x)}$

b) Estimate: $C(37)-C(36)\approx C'(36)=\frac{1}{4}\cdot 36+3=\boxed{12}$

Actual cost: $C(37)-C(36)=380.125-368=\boxed{\$12.13}$

c)

Estimate: $R'(36)=\frac{1}{3}(75-2\cdot 36)=\frac{1}{3}(75-72)=\boxed{\$1}$

Actual revenue: $R(37)-R(36)=\frac{1}{3}(75-37)\cdot 37-\frac{1}{3}(75-36)36$

$=468.67-468=\boxed{\$0.67}$

Appoximations

If $f(x)$ is differentiable at $x=x_0$ and Δx is a small change in x, then

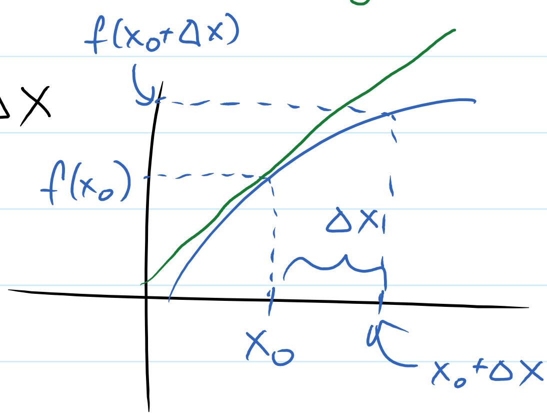
$$f(x_0+\Delta x)\approx f(x_0)+f'(x_0)\Delta x,$$

$y-f(x_0)=f'(x_0)(x-x_0)$

$y=f'(x_0)(x-x_0)+f(x_0)$

or equivalently

$$\Delta f\approx f'(x_0)\Delta x$$



Find Δf if $\Delta x=1$,

$\Delta f=f(x_0+\Delta x)-f(x_0)$

$=f(x_0+1)-f(x_0)\approx f'(x_0)$

EXAMPLE 2.5.3 Estimating Change in Cost Using a Derivative

Suppose the total cost of manufacturing q hundred units of a certain commodity is C thousand dollars where $C(q) = 3q^2 + 5q + 10$. If the current level of production is 4,000 units, estimate how the total cost will change if 4,050 units are produced.

$q = 40$ $\qquad \Delta C$

$C(q) = 3q^2 + 5q + 10$

estimate: $C(40.5) - C(40)$

$\Delta C = C'(x_0) \cdot \Delta x$

$\Delta x = \text{new} - \text{current} = 40.5 - 40 = 0.5$

$C'(q) = 6q + 5$

$C(40.5) - C(40) \approx \Delta C = [6 \cdot (40) + 5] \cdot 0.5$

$= (240 + 5)\frac{1}{2} = 245/2 = \boxed{122.5}$

$\boxed{\$122500}$

EXAMPLE 2.5.4 Estimating Error in Measurement

During a medical procedure, the size of a roughly spherical tumor is estimated by measuring its diameter and using the formula $V = \frac{4}{3}\pi R^3$ to compute its volume. If the diameter is measured as 2.5 cm with a maximum error of 2%, how accurate is the volume measurement?

$r = \frac{d}{2} = 1.25$

$V(\overset{1.25}{\cancel{2.5}}) = \frac{4}{3} \cdot \pi \cdot (1.25)^3 = 8.18 \text{ cm}^3$

error in measurement: 2% of $\overset{1.25}{\cancel{2.5}}$ cm is $\overset{0.025}{\cancel{0.05}}$

measured: $(\cancel{2.45}, \cancel{2.55})$ $(1.225, 1.275)$

exact volume is $(V(\overset{1.225}{\cancel{2.45}}), V(\overset{1.275}{\cancel{2.55}}))$

estimate: $V(\overset{1.275}{\cancel{2.55}}) - V(\overset{1.25}{\cancel{2.5}}) \approx V'(\overset{1.25}{\cancel{2.5}}) \cdot \underbrace{\Delta R}_{}$ $\overset{0.025}{\cancel{0.05}}$

$V(R) = \frac{4}{3}\pi R^3 \qquad V'(R) = \frac{4}{3}\pi 3R^2 = 4\pi R^2$

estimate: $V(\cancel{2.55}) - V(\cancel{2.5}) \approx V'(\cancel{2.5}) \cdot \Delta R$

$V(R) = \frac{4}{3}\pi R^3 \qquad V'(R) = \frac{4}{\cancel{3}}\pi \cancel{3} R^2 = 4\pi R^2$

$$V'(\overset{1.25}{\cancel{2.5}}) \cdot 0.05 = 4 \cdot \pi \, (\overset{1.25}{\cancel{2.5}})^2 \cdot \overset{0.025}{\cancel{0.05}} = 0.49$$

$\Delta V = 0.49$

$V(1.25) = 8.18$

estimate for the volume is $(8.18 - 0.49, 8.18 + 0.49)$

$$\boxed{(7.69, 8.67)}$$

Differentials

The differential of x is $dx = \Delta x$,
and if $y = f(x)$ is a differentiable function,
then $dy = f'(x)\,dx$ is the differential of y.

Ex: Find dy if:

a) $f(x) = x^3 - 7x^2 + 2$

$$dy = f'(x)\,dx = (3x^2 - 14x)\,dx$$

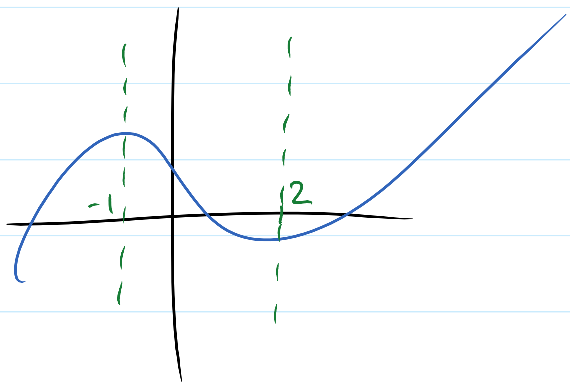
b) $f(x) = (x^2 + 5)(3 - x - 2x^2)$

$(f \cdot g)' = g f' + f g'$

$$dy = \left[(3 - x - 2x^2) \cdot 2x + (x^2 + 5)(-1 - 4x)\right] dx$$

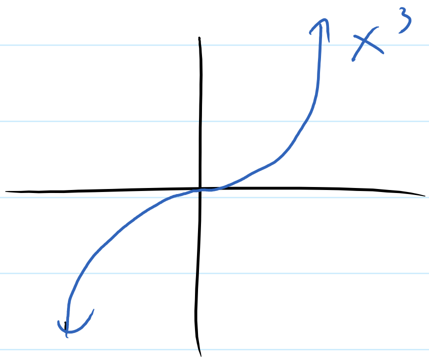
Section 3.1

Section 3.1



A differentiable function is increasing at c, if $f'(c) > 0$, it is decreasing if $f'(c) < 0$.

Def: The value c is called a critical value (or point) if $f'(c) = 0$ or $f'(c)$ DNE. ($f(c)$ exists)



Maximum or a minimum of a function is at a critical point.

Ex: Find inc/dec intervals:

$$f(x) = 2x^3 + 3x^2 - 12x - 7$$

$$f'(x) = 6x^2 + 6x - 12 = 0$$

$$6(x^2 + x - 2) = 0$$

$$6(x-1)(x+2) = 0$$

$$x = 1, -2$$

f'	c	$f'(c)$	inc/dec
$(-\infty, -2)$	-3	$6(-3-1)(-3+2) = + \cdot - \cdot - = +$	inc $\nearrow$
$(-2, 1)$	0	$6(-1)(+2) = -$	dec $\searrow$
$(1, \infty)$	3	$6(3-1)(3+2) = +$	inc $\nearrow$