MAC2311, Summer 2015

Exam #2

June 19, 2015

Name Key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No graphing calculators are allowed!

3.1-11

1. (10 points) Find the point (x, y), at which the graph $y = 3x^2 + 3x - 10$ has a horizontal tangent.

$y' = 6x + 3$

$y' = 0$

$6x + 3 = 0$

$x = \frac{-3}{6} = \frac{-1}{2}$

$y(-\frac{1}{2}) = 3 \cdot \frac{1}{4} + 3 \cdot (\frac{-1}{2}) - 10$

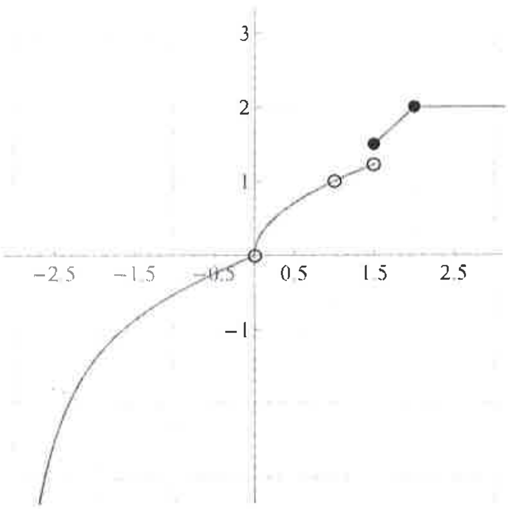
$= \frac{3}{4} - \frac{3}{2} - 10 = \frac{3 - 6 - 40}{4}$

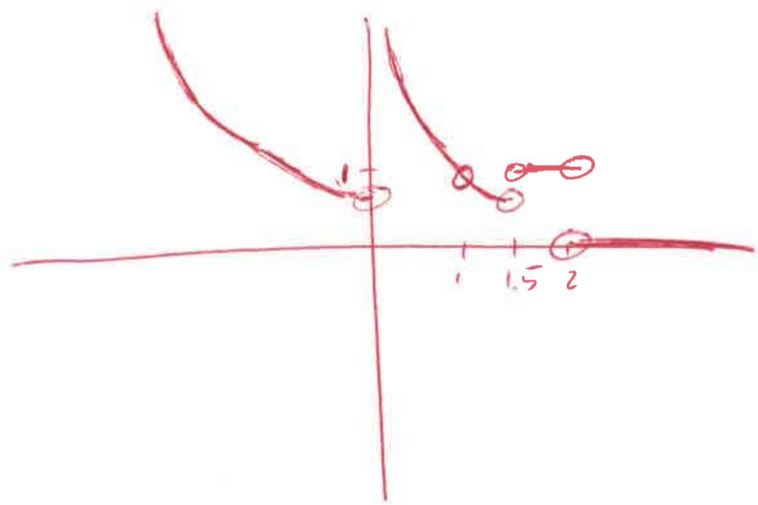
$= \frac{-43}{4}$

$\boxed{(-\frac{1}{2}, \frac{-43}{4})}$

3.2-17

2. (10 points) Graph the derivative of the function graphed below.





3. (5 points) The derivative of a function represents an instantaneous rate of change of the function with respect to its variable. (true/false)

$\boxed{3.3\text{-}18}$

4. (10 points) Find the first and second derivatives.

a) $y = 6x^3 + 5x - 6x^{-3}$

$y' = 18x^2 + 5 - 6\cdot(-3)x^{-4} = \boxed{18x^2 + 5 + 18x^{-4}}$

$y'' = 18\cdot 2x + 18(-4)x^{-5} = \boxed{36x - 72x^{-5}}$

b) $y = e^{2x^2}$ $\boxed{3.3\text{-}14}$

$y' = e^{2x^2}\cdot(2x^2)' = e^{2x^2}\cdot 2\cdot 2x = \boxed{4e^{2x^2}\cdot x}$

$y'' = 4\cdot\left((e^{2x^2})'x + 1\cdot e^{2x^2}\right) = 4\cdot\left[4e^{2x^2}\cdot x\cdot x + e^{2x^2}\right]$

$\boxed{= 4e^{2x^2}(4x^2+1)} = 16e^{2x^2}x^2 + 4e^{2x^2}$

5. (10 points) Find the first derivative of the function. $\boxed{3.3\text{-}10}$

a) $y = (3-t)(1+t^2)^{-1}$

$y' = (3-t)'(1+t^2)^{-1} + (-1)(1+t^2)^{-2}\cdot(1+t^2)'\cdot(3-t)$

$\boxed{= -(1+t^2)^{-1} - (1+t^2)^{-2}\cdot 2t(3-t)}$

$= (1+t^2)^{-2}\left[-(1+t^2) - 2t(3-t)\right] = (1+t^2)^{-2}\left[-1-t^2-6t+2t^2\right]$

$\boxed{= (1+t^2)^{-2}\left[t^2-6t-1\right]}$

$\boxed{3.4-4}$

6. (10 points) At time t, the position of a body moving along the s-axis is $s = -t^3 + 9t^2 - 24t$ m.

   a) Find the body's acceleration each time the velocity is zero.

   b) Find the body's speed each time the acceleration is zero.

velocity: $s' = -3t^2 + 18t - 24 = -3(t^2 - 6t + 8) = -3(t-2)(t-4)$

accel: $s'' = -3(2t-6) = -6(t-3)$

a) velocity is zero at $t=2$ and $t=4$

$s''(2) = -6(2-3) = \boxed{6}\ m/s^2$

$s''(4) = -6(4-3) = \boxed{-6}\ m/s^2$

b) acc. is zero at $t=3$

$s'(3) = -3(3-2)(3-4) = \boxed{3}\ m/s$

7. (10 points) Find the first and the second derivative of the function. $\boxed{3.5-18}$

$$s = 7 \sec t$$

$s' = 7 \cdot \sec t \cdot \tan t$

$s'' = 7 \cdot (\sec t \cdot \tan t \cdot \tan t + \sec^2 t \cdot \sec t)$

$\boxed{= 7(\sec t \cdot \tan^2 t + \sec^3 t)}$

$\boxed{= 7 \sec t (\tan^2 t + \sec^2 t)}$

8. (10 points) Use implicit differentiation to find the derivative of y. $\boxed{\text{Quiz}}$

$$3x^2y + y^2 = x + y$$

$$3 \cdot 2xy + 3x^2y' + 2yy' = 1 + y'$$

$$3x^2y' + 2yy' - y' = 1 - 3 \cdot 2xy$$

$$\boxed{y' = \frac{1 - 6xy}{3x^2 + 2y - 1}}$$

9. (5 points each) Find the derivative of y. $\boxed{3.8\text{-}13}$

a) $y = \dfrac{\ln x}{e^x + \ln x}$

$$y' = \frac{\frac{1}{x}(e^x + \ln x) - (e^x + \frac{1}{x})\ln x}{(e^x + \ln x)^2} = \frac{\frac{e^x}{x} + \cancel{\frac{\ln x}{x}} - e^x \ln x - \cancel{\frac{\ln x}{x}}}{(e^x + \ln x)^2}$$

$$\boxed{= \frac{e^x\left(\frac{1}{x} - \ln x\right)}{(e^x + \ln x)^2}}$$

b) $y = \sin^{-1}(3x^2 + 3)$ $\boxed{3.9\text{-}13}$

$$y' = \frac{1}{\sqrt{1 - (3x^2+3)^2}} \cdot (3 \cdot 2x) = \boxed{\frac{6x}{\sqrt{1 - (3x^2+3)^2}}}$$

10. (5 points) Use logarithmic differentiation to find the derivative of y. Do not simplify your answer.

$$y = (x+3)^3(x^2-9)^2(1-x) \quad \boxed{3.8-23}$$

$$\ln y = 3\ln(x+3) + 2\ln(x^2-9) + \ln(1-x)$$

$$\frac{1}{y}y' = \frac{3}{x+3} + \frac{2\cdot(2x)}{x^2-9} + \frac{-1}{1-x}$$

$$y' = y\left(\frac{3}{x+3} + \frac{4x}{x^2-9} - \frac{1}{1-x}\right) = \boxed{(x+3)^3(x^2-9)^2(1-x)\left(\frac{3}{x+3} + \frac{4x}{x^2-9} - \frac{1}{1-x}\right)}$$

11. (10 points) Use logarithmic differentiation to find the derivative of y.

$$y = (\sin x)^x \quad \boxed{3.8-38}$$

$$\ln y = x \cdot \ln(\sin x)$$

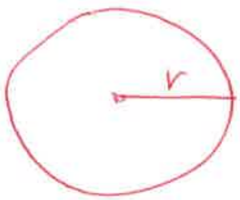
$$\frac{1}{y}y' = \ln(\sin x) + \frac{1}{\sin x} \cdot \cos x \cdot x$$

$$y' = y\left(\ln(\sin x) + \frac{\cos x}{\sin x} \cdot x\right)$$

$$\boxed{= (\sin x)^x\left(\ln(\sin x) + x \cdot \cot x\right)}$$

3.10-8

12. (10 extra points) When a circular plate of metal is heated in an oven, its radius increases at a rate of 0.03cm/min. At what rate is the plate's area increasing when the radius is 73 cm?



$r' = \frac{dr}{dt} = 0.03\ \frac{cm}{min}$, $r = 73$ cm

Find $A' = \frac{dA}{dt}$

$A = \pi r^2$

$A' = \pi 2 r r'$

$A' = 2 \cdot \pi \cdot 73 \cdot 0.03 = \boxed{13.76} \frac{cm^2}{min}$

$= \boxed{4.38 \cdot \pi}$

The area is increasing at the rate of $13.76 \frac{cm^2}{min}$