SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

C(x) is the total cost of producing *x* units of a particular commodity and *p(x)* is the unit price at which all *x* units will be sold. Assume *p(x)* and *C(x)* are in dollars. Find the marginal cost and the marginal revenue.

1) $C(x) = \frac{1}{4}x^2 + 5x + 53; \; p(x) = \frac{1}{3}(36 - x)$

C(x) is the total cost of producing *x* units of a particular commodity. Assume *C(x)* is in dollars. Use marginal cost to estimate the cost of producing the 21st unit. What is the actual cost of producing the 21st unit?

2) $C(x) = \frac{1}{4}x^2 + 4x + 65$

Use increments to make the required estimate.

3) Estimate how much the function $f(x) = x^2 - 3x + 3$ will change as *x* increases from 5 to 5.3.

Solve the problem.

4) A manufacturer's total cost is $C(q) = 0.2q^3 - 0.2q^2 + 500q + 230$ when *q* thousand units are produced. Currently, 4000 units ($q = 4$) are being produced and the manufacturer is planning to increase the level of production to 4100. Use marginal analysis to estimate how this change will affect total cost.

Specify the intervals on which the derivative of the given function is positive and those on which it is negative.

5)
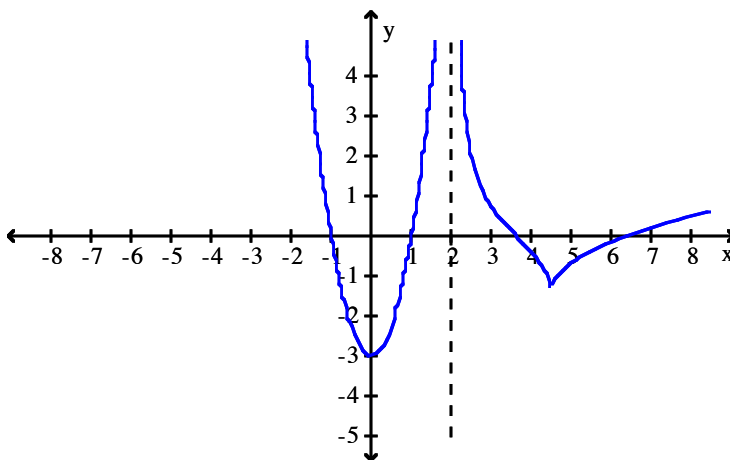
Find the intervals of increase and decrease for the given function.

6) $f(x) = x^2 + 4x + 3$

7) $f(x) = x^3 - 27x - 7$

8) $f(t) = \dfrac{1}{49 - t^2}$

9) $f(t) = \dfrac{t}{(t + 3)^2}$

Determine the critical numbers of the given function and classify each critical point as a relative maximum, a relative minimum, or neither.

10) $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$

11) $f(t) = \dfrac{t}{t^2 + 11}$

12) $g(x) = 4 - \dfrac{2}{x} + \dfrac{9}{x^2}$

Use calculus to sketch the graph of the given function.

13) $f(x) = x^3 - 3x^2$

14) $f(x) = 3x^4 + 8x^3 + 6x^2 - 3$

15) $f(x) = x^3(x + 4)^2$

16) $g(t) = \dfrac{t}{t^2 + 5}$

The derivative of a function $f(x)$ is given. In each case, find the critical numbers of $f(x)$ and classify each as corresponding to a relative maximum, a relative minimum, or neither.

17) $f'(x) = x^2(9 - x^2)$

Determine where the graph of the given function is concave upward and concave downward. Find the coordinates of all inflection points.

18) $f(x) = x^3 + 6x^2 + x + 9$

19) $g(t) = t^2 - \dfrac{27}{t}$

Use first and second derivative information to sketch the graph of the function.

20) $f(x) = \dfrac{1}{3}x^3 - 9x + 3$

21) $f(x) = 2x^5 - 10x - 7$

22) $g(x) = \sqrt{x^2 + 9}$

23) $f(x) = \dfrac{2}{x^2 + x + 7}$

Use the second derivative test to find the relative maxima and minima of the given function.

24) $f(x) = x^4 - 32x^2 + 7$

25) $f(x) = 2x + 9 + \dfrac{8}{x}$

26) $h(x) = \dfrac{5}{1 + x^2}$

The second derivative $f''(x)$ of a function is given. In each case, use this information to determine where the graph of $f(x)$ is concave upward and concave downward and find all values of x for which an inflection point occurs. [You are not required to find $f(x)$ or the y coordinates of the inflection points.]

27) $f''(x) = x^2(x - 7)(x - 5)$

Solve the problem.

28) Sketch the graph of a function that has all the following properties:
   a. $f'(x) > 0$ when $x < -1$ and when $x > 1$
   b. $f'(x) < 0$ when $-1 < x < 1$
   c. $f''(x) < 0$ when $x < 0$
   d. $f''(x) > 0$ when $x > 0$

Find all vertical and horizontal asymptotes of the graph of the given function.

29) $f(x) = \dfrac{6x - 9}{x + 2}$

30) $f(t) = \dfrac{t + 5}{t^2}$

Sketch the graph of the given function.

31) $f(x) = x^4 + 4x^3 + 4x^2 - 1$

32) $f(x) = x - \dfrac{1}{x}$

3

33) $f(x) = \dfrac{x^2 - 9}{x^2 + 1}$

Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

34) $f(x) = x^3 - 6x^2 + 7; \; -1 \le x \le 6$

35) $f(x) = x^5 - 20x^4 + 8; \; 0 \le x \le 17$

36) $f(t) = \dfrac{t^2}{t - 8}; \; -2 \le t \le -\dfrac{1}{4}$

37) $f(x) = \dfrac{1}{x + 12}; \; x \ge 0$

Compute the elasticity of demand for the given demand function $D(p)$ and determine whether the demand is elastic, inelastic, or of unit elasticity at the indicated price p.

38) $D(p) = 150 - p^2; \; p = 9$

39) $D(p) = \dfrac{2300}{p^2}; \; p = 8$

Solve the problem.

40) When a particular commodity is priced at p dollars per unit, consumers demand q units, where p and q are related by the equation $q^2 + 5pq = 24$.

   a. Find the elasticity of demand for this commodity.
   b. For a unit price of $2, is the demand elastic, inelastic, or of unit elasticity?

41) An airline determines that when a round-trip ticket between Los Angeles and San Francisco costs p dollars ($0 \le p \le 120$), the daily demand for tickets is $q = 432 - 0.03p^2$.
   a. Find the elasticity of demand. Determine the values of p for which the demand is elastic, inelastic, and of unit elasticity.
   b. Interpret the results of part (**a**) in terms of the behavior of the total revenue as a function of unit price p.

42) A store has been selling a popular computer game at the price of $44 per unit, and at this price, players have been buying 72 units per month. The owner of the store wishes to raise the price of the game and estimates that for each $1 increase in price, three fewer units will be sold each month. If each unit costs the store $26, at what price should the game be sold to maximize profit?

43) It is estimated that the cost of constructing an office building that is n floors high is
$C(n) = 3n^2 + 300n + 700$ thousand dollars. How many floors should the building have to minimize the average cost per floor? (Remember that your answer should be a whole number.)

44) Find the present value of $17,000 over a term of 4 years at an annual interest rate of 8% if interest is compounded:
a. Annually
b. Quarterly

45) Find the present value of $16,000 over a term of 8 years at an annual interest rate of 7% if interest is compounded continuously.

46) Bob and Alice want to remodel their bathroom in 3 years. They estimate the job will cost $29,000. How much must they invest now at an annual interest rate of 6% compounded quarterly to achieve their goal?

47) The Morenos invest $9000 in an account that grows to $11,000 in 4 years. What is the annual interest rate r if interest is compounded
a. Quarterly   **b.** Continuously

48) How quickly will money double if it is invested at an annual interest rate of 10% compounded continuously?

Differentiate the given function.

49) $f(x) = 9e^{9x + 7}$

50) $f(x) = \dfrac{3e^x}{2x}$

51) $f(x) = xe^{-x^3}$

52) $f(x) = x^9 \ln x$

53) $f(x) = e^{-5x} + x^4$

54) $f(x) = \ln(9x^3 + 8x - 5)$

55) $f(t) = \sqrt{6\ln t + 7t}$

56) $f(x) = x^6 5^{x^5}$

Find the largest and smallest values of the given function over the prescribed closed, bounded interval.

57) $F(x) = 3e^{(3x^2 - 6x)}$ for $0 \le x \le 2$

58) $g(x) = \dfrac{e^x}{5x + 1}$ for $0 \le x \le 1$

Find the second derivative of the given function.

59) $f(x) = 4e^{2x} + 9e^{-x}$

Find an equation for the tangent line to $y = f(x)$ at the specified point.

60) $f(x) = -5xe^x$, where $x = 0$

61) $f(x) = x - 5\ln x$, where $x = e$

Use logarithmic differentiation to find the derivative $f'(x)$.

62) $f(x) = (3x + 4)^2(x - 4x^2)^{1/2}$

63) $f(x) = \sqrt[3]{\dfrac{2x + 1}{1 - 5x}}$

Solve the problem.

64) Paul Edwards owns an electronics firm. He determines that when he employs x thousand people, the profit will be P million dollars, where

$$P(x) = \frac{2}{5}\ln(5x + 2) + 3x - x^2$$

How many workers should Paul employ to maximize profit? What is the maximum profit?

6

1) marginal cost $= \frac{1}{2}x + 5$;

   marginal revenue $= 12 - \frac{2x}{3}$

2) estimated cost $= \$14.00$;
   actual cost $= \$14.25$

3) 2.1

4) The approximate change in
   cost will be $50.80.

5) $f'(x) > 0$ for $0 < x < 2$ and
   $x > 4.5$;
   $f'(x) < 0$ for $x < 0$ and $2 < x < 4.5$

6) $f(x)$ is increasing for $x > -2$; $f(x)$ is decreasing for $x < -2$.

7) $f(x)$ is increasing for $x < -3$
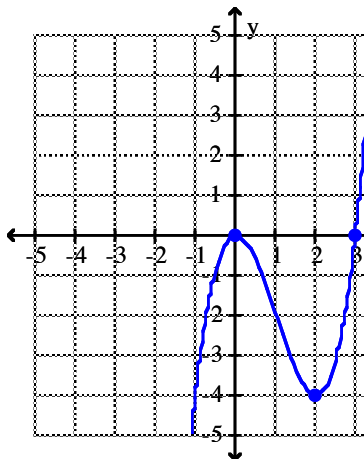   and $x > 3$; $f(x)$ is
   decreasing for $-3 < x < 3$.

8) $f(t)$ is increasing for $0 < t < 7$ and $t > 7$; $f(t)$ is
   decreasing for $t < -7$ and $-7 < t < 0$.

9) $f(t)$ is increasing on $-3 < t < 3$; $f(t)$ is decreasing on $t < -3$ and $t > 3$.

10) $x = 0, 1$; $(0, 1)$ relative
    minimum; $(1, 2)$ neither

11) $t = -\sqrt{11}, \sqrt{11}$;
    $\left(-\sqrt{11}, -\frac{\sqrt{11}}{22}\right)$ relative

    minimum; $\left(\sqrt{11}, \frac{\sqrt{11}}{22}\right)$

    relative maximum

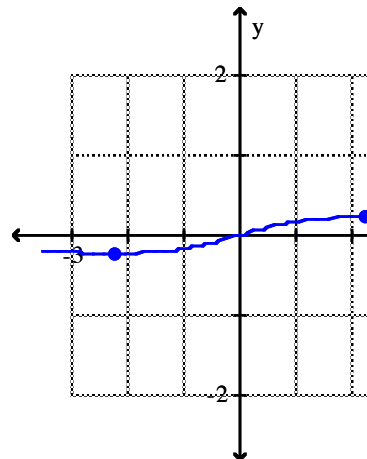12) $x = 9$; $\left(9, \frac{35}{9}\right)$ relative

    minimum

13)
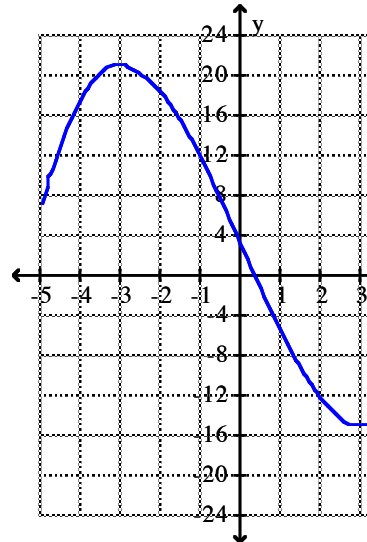

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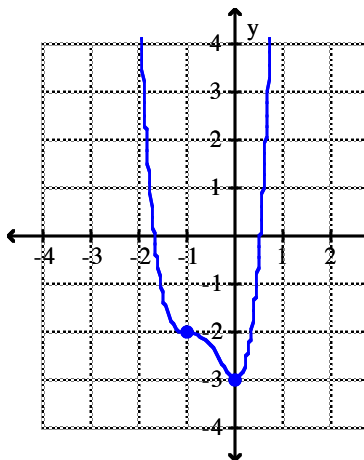
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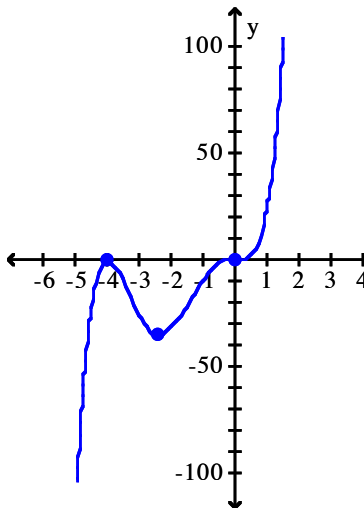
16)

17)

Critical Numbers	Cla
-3	Relat
0	
3	Relati

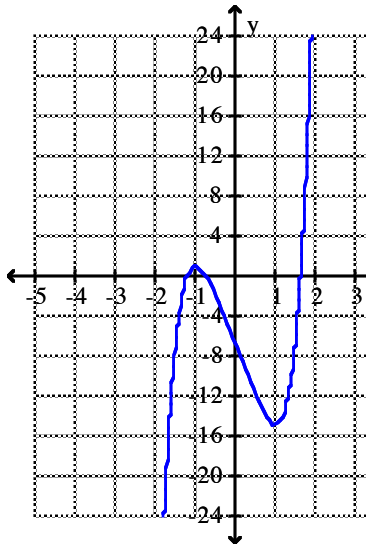18) Concave upward for $x > -2$;
    concave downward for
    $x < -2$; inflection at $(-2, 11)$

19) upward for $t < 0$ and $t > 3$;
    downward for $0 < t < 3$;
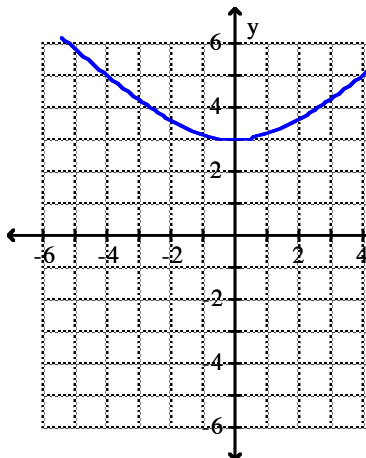   inflection at $(3, 0)$
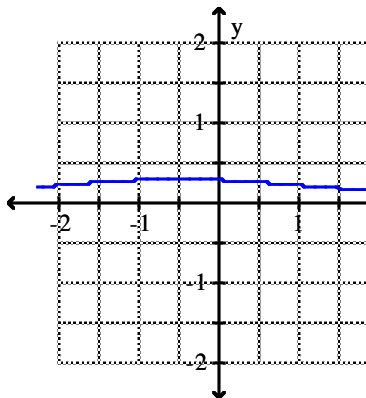
20)

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21)

22)

23)

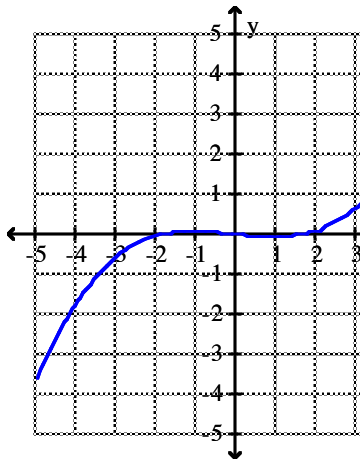

24) min at (-4, -249); max at (0, 7); min at (4, -249)

25) max at (-2, 1); min at (2, 17)

26) max at (0, 5)

27) Concave upward for $x < 0$, for $0 < x < 5$, and for $x > 7$; concave downward for $5 < x < 7$; inflection points at $x = 5, 7$
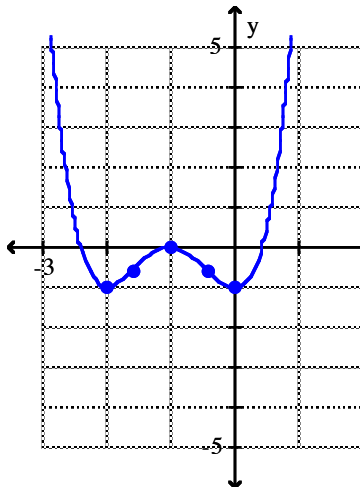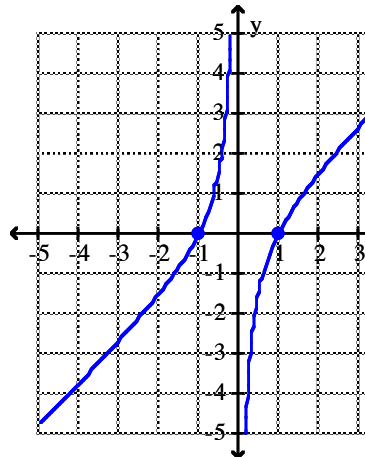
28)

29) Vertical asymptote, $x = -2$; horizontal asymptote, $y = 6$

30) Vertical asymptote, $t = 0$; horizontal asymptote, $y = 0$
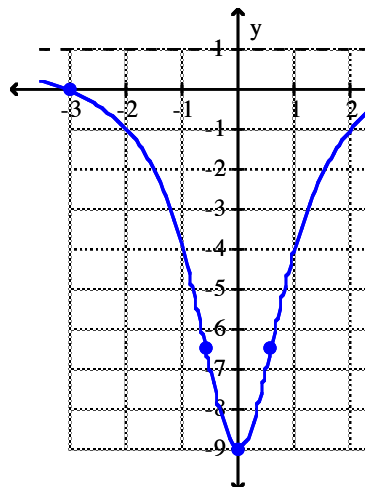
31)

32)

33)

34) min = -25; max = 7

35) min = -262,136; max = 8

36) min = $-\dfrac{2}{5}$; max = $-\dfrac{1}{132}$

37) no min; max = $\dfrac{1}{12}$

38) $E(p) = \dfrac{2p^2}{150 - p^2}$; $E(9) \approx$ 2.348, elastic

39) $E(p) = 2$; $E(8) = 2$, elastic

40) **a.** $E = \dfrac{5p}{2q + 5p}$

b. When $p = 2$, demand is inelastic.

8

41) **a.** $E(p) = \dfrac{0.02p^2}{144 - 0.01p^2}$

Demand is elastic when $p >$ 69.28, inelastic when $p <$ 69.28, and of unit elasticity when $p = 69.28$.
b. If a ticket costs more than $69.28 then revenue is decreasing as the price increases. If the cost is less than $69.28 then revenue increases with price. If the price equals $69.28 then revenue is unaffected by a small change in price.

42) $47

43) 15 floors

44) **a.** $12,495.51
   b. $12,383.58

45) $\approx$ $9139.35

46) $\approx$ $24,255.24

47) **a.** $\approx 5.048\%$
   b. $\approx 5.017\%$

48) ≈ 6.93 years

49) $f'(x) = 81e^{9x+7}$

50) $f'(x) = \dfrac{3e^x(x-1)}{2x^2}$

51) $f'(x) = e^{-x^3}(1 - 3x^3)$

52) $f'(x) = x^8(9\ln x + 1)$

53) $f'(x) = -5e^{5x} + 4x^3$

54) $f'(x) = \dfrac{27x^2 + 8}{9x^3 + 8x - 5}$

55) $f'(t) = \dfrac{7t + 6}{2t\sqrt{6\ln t + 7t}}$

56) $6x^5 5^{x^5} + 5x^{10}(\ln 5)5^{x^5}$

57) smallest: $3e^3$; largest: 3

58) smallest: $\dfrac{1}{5}e^{4/5}$; largest: 1

59) $f''(x) = 16e^{2x} + 9e^x$

60) $y = -5x$

61) $y = (1 - \dfrac{5}{e})x$

62) $f'(x) = f(x)$
$\left[\dfrac{6}{3x + 4} + \dfrac{1 - 8x}{2(x - 4x^2)} \right]$

63) $f'(x) = f(x)\left(\dfrac{1}{3}\right)$
$\left(\dfrac{2}{2x + 1} + \dfrac{5}{1 - 5x} \right)$

64) 1600 people; $\approx$ $3,161,034