MAC 2233, Fall 2017

Exam #2

October 24, 2017

Name Key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No calculators are allowed!

Revenue function:	$R(x) = p * x$
Profit function:	$P(x) = R(x) - C(x)$
Elasticity of demand:	$E(p) = -\frac{p \cdot q'(p)}{q(p)}$
Future value of an investment:	$B(t) = P(1 + \frac{r}{k})^{kt}$ $B(t) = Pe^{rt}$
Effective interest:	$r_e = (1 + \frac{r}{k})^k - 1$ $r_e = e^r - 1$

Honor Code: On my honor, I have neither received nor given any aid during this examination.

Signature: ______________________

1. (15 points) Find the intervals where the function is increasing/decreasing, concave up/down and find the relative min/max and inflection points.

$$f(x) = \frac{x}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)\cdot 1 - x\cdot 2x}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$1-x^2=0$
$1=x^2$
$\underline{x=\pm 1}$

$x^2+1=0$
$x^2=-1$
$\underline{\text{none}}$

f'	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
$1-x^2$	$-$	$+$	$-$
$(x^2+1)^2$	$+$	$+$	$+$
f'	$-$ $\searrow$	$+$ $\nearrow$	$-$ $\searrow$

$$\boxed{\begin{array}{l} x=-1 \text{ rel. min} \\ x=1 \text{ rel. max} \end{array}}$$

$$f'(x) = \frac{1-x^2}{(x^2+1)^2}$$

$$f''(x) = \frac{(x^2+1)^2\cdot(-2x) - (1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4} = \frac{-2x(x^2+1)^2 - 4x(x^2+1)(1-x^2)}{(x^2+1)^4}$$

$$= \frac{2x\,\cancel{(x^2+1)}\left[-(x^2+1)-2(1-x^2)\right]}{(x^2+1)^{\cancel{4}\,3}} = \frac{2x(-x^2-1-2+2x^2)}{(x^2+1)^3}$$

$$= \frac{2x(x^2-3)}{(x^2+1)^3}$$

$2x=0$
$\underline{x=0}$

$x^2-3=0$
$\underline{x=\pm\sqrt{3}}$

f''	$(-\infty,-\sqrt{3})$	$(-\sqrt{3},0)$	$(0,\sqrt{3})$	$(\sqrt{3},\infty)$
$2x$	$-$	$-$	$+$	$+$
x^2-3	$+$	$-$	$-$	$+$
$(x^2+1)^3$	$+$	$+$	$+$	$+$
f''	$-$ $\frown$	$+$ $\smile$	$-$ $\frown$	$+$ $\smile$

$\boxed{x=\pm\sqrt{3},0}$ are inflection pts

2. (10 points) Find the critical numbers of the given function and classify each as a relative minimum or maximum

$$f(x) = x^3(x-2)^2$$

$f'(x) = 3x^2(x-2)^2 + 2(x-2)x^3 = x^2(x-2)\left[3(x-2)+2x\right]$

$= x^2(x-2)(3x-6+2x) = x^2(x-2)(5x-6)$

$f'(x) = 0$ $\quad x^2=0$ $\quad x-2=0$ $\quad 5x-6=0$

$x=0$ $\quad x=2$ $\quad x=6/5$

$\boxed{x = \frac{6}{5} \text{ is a relative max}}$

$\boxed{x=0, \frac{6}{5}, 2 \text{ are crit. num}}$

$\boxed{x=2 \text{ is a rel. min}}$

f'	$(-\infty,0)$	$(0,\frac{6}{5})$	$(\frac{6}{5},2)$	$(2,\infty)$
x^2	+	+	+	+
$x-2$	−	−	−	+
$5x-6$	−	−	+	+
f'	+	+	−	+

3. (10 points each) Find the intervals where the function is increasing/decreasing

(a) $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12 = 0$

$3(x^2-4) = 0$

$3(x-2)(x+2) = 0$

$x = \pm 2$

f'	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
	+	−	+
	↗	↘	↗

(b) $f(\cancel{t}\,x) = \frac{16}{x} + x^2$ $\quad f'(x) = -16x^{-2} + 2x = 2x^{-2}(-8+x^3)$

$= 16x^{-1} + x^2$

$x=0$ $\quad x^3 = 8$

$x = 2$

f'	$(-\infty,0)$	$(0,2)$	$(2,\infty)$
$\frac{2}{x^2}$	+	+	+
x^3-8	−	−	+
f'	− ↘	− ↘	+ ↗

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4. (10 points) Find the elasticity of demand and determine whether the demand is elastic, inelastic, or unitary at the indicated price.

(a) $q(p) = 240 - 2p; p = 50$

$$q'(p) = -2 \qquad E(p) = \frac{2p}{240-2p} = \frac{p}{120-p}$$

$$E(50) = \frac{50}{70} = \boxed{\frac{5}{7}} < 1$$

Demand is inelastic.

(b) $q(p) = 300 - p^2; p = 10$

$$q'(p) = -2p \qquad E(p) = \frac{2p^2}{300-p^2}$$

$$E(10) = \frac{2 \cdot 100}{300-100} = \frac{200}{200} = \boxed{1}$$

Demand is unitary.

5. (10 points) Differentiate the given function.

(a) $f(x) = e^{3x+1}$

$$f'(x) = e^{3x+1} \cdot (3x+1)' = e^{3x+1} \cdot 3$$

$$= \boxed{3e^{3x+1}}$$

(b) $f(x) = 3\log_5 x^2$

$$f(x) = 3 \cdot 2 \log_5 x = 6 \log_5 x$$

$$f'(x) = \boxed{\frac{6}{x \cdot \ln 5}}$$

6. (10 points) Use the marginal cost to estimate the cost of producing the 6th unit of a commodity if the cost function is

$$C(x) = \frac{1}{2}x^2 - 3x + 110$$

$C'(x) = x - 3$

$C'(5) = 5 - 3 = \boxed{2}$

7. (15 points) A citrus grower in Florida estimates that if 100 orange trees are planted, the average yield will be 60 oranges per tree. The average yield will decrease by 2 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plan to maximize the total yield? [Hint: Use x to denote the number of orange trees and find the total yield as a function of x.]

x	yield per tree	total yield
100	60	$100 \cdot 60$
101	$60-2$	$101 \cdot (60-2)$
102	$60 - 2\cdot(102-100)$	$102 \cdot (60 - 2(102-100))$
x	$60-2(x-100) = 60-2x+200$ $260-2x$	$x(60-2(x-100)) = x(260-2x) = 260x-2x^2$

$T(x) = 260x - 2x^2$

$T''(x) = -4$

$T'(x) = 260 - 4x = 0$

$x = \frac{260}{4} = \boxed{65}$

Hence $x=65$ is a relative maximum.

8. (10 points) Differentiate [Hint: simplify first]

$$f(x) = \ln\left(x^7(x^2+3)^4\right)$$

$$= \ln x^7 + \ln (x^2+3)^4$$

$$= 7\ln x + 4\ln(x^2+3)$$

$$f'(x) = \frac{7}{x} + 4 \cdot \frac{2x}{x^2+3} = \boxed{\frac{7}{x} + \frac{8x}{x^2+3}}$$

9. (5 extra credit points) Find the derivative of $f(x) = x^x$.

$$x^x = e^{x\ln x}$$

$$f'(x) = e^{x\cdot\ln x} \cdot (x\cdot\ln x)' = x^x\left(\ln x + x\cdot\frac{1}{x}\right)$$

$$= \boxed{x^x(1+\ln x)}$$

10. (5 extra credit points) Find the absolute minimum and maximum of the function $f(x) = \dfrac{e^x}{x}$ in the interval $[1, 4]$. [Hint: $e^4 \approx 54.6$]

$$f'(x) = \frac{e^x\cdot x - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$$

$e^x = 0$ none $\qquad x-1=0$, $x=1$ $\qquad x^2=0$, $x=0$

Since only $x=1$ is in $[1,4]$ we have to compare $f(1)$ and $f(4)$.

$$f(1) = \frac{e^1}{1} = e \approx 2.718$$

$$f(4) = \frac{e^4}{4} \approx \frac{54}{4} \approx 13$$

$x=4$ is abs. max

$x=1$ is abs. min